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Wage Dynamics and Returns to Unobserved Skill

by

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Abstract

Economists disagree about the factors driving the substantial increase in residual wage inequality in the U.S. over the past few decades. We identify and estimate a general model of log wage residuals that incorporates: (i) changing returns to unobserved skills, (ii) a changing distribution of unobserved skills, and (iii) changing volatility in wages due to factors unrelated to skills. Using data from the Panel Study of Income Dynamics, we estimate that the returns to unobserved skills have declined by as much as 50% since the mid-1980s despite a sizeable increase in residual inequality. Instead, the variance of skills rose over this period because of increasing variability in lifecycle skill growth. Finally, we develop an assignment model of the labor market and show that both demand and supply factors contributed to the downward trend in the returns to skills over time, with demand factors dominating for non-college men.

Bank topics: Econometric and statistical methods; Labour markets
JEL codes: C23, J24, J31

Résumé

Les économistes ne s'entendent pas sur le choix des facteurs qui sont à l'origine de l'augmentation importante des inégalités résiduelles de salaire observée aux États-Unis depuis les dernières décennies. Nous identifions et estimons les paramètres d'un modèle général des valeurs résiduelles du logarithme des salaires qui intègre : 1) des rendements fluctuants des compétences non observées, 2) des distributions changeantes des compétences non observées, et 3) des changements dans la volatilité des salaires causés par des facteurs non liés aux compétences. À partir des données de panel de l'étude sur la dynamique des revenus, nous estimons que le rendement des compétences non observées a enregistré un recul allant jusqu'à 50 % depuis le milieu des années 1980 en dépit d'une hausse appréciable des inégalités résiduelles. La variance des compétences a plutôt augmenté au cours de la période en raison d'une plus grande variabilité de la croissance des compétences durant le cycle de vie. Enfin, nous élaborons un modèle d'affectation de la main-d'œuvre sur le marché du travail et montrons que les facteurs liés à la demande et à l'offre ont ensemble contribué à la tendance à la baisse du rendement des compétences au fil du temps, les facteurs associés à la demande étant prépondérants chez les hommes qui n'ont pas fait d'études post-secondaires.

Sujets : Méthodes économétriques et statistiques; Marchés du travail
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Non-Technical Summary

The U.S. has experienced substantial and sustained growth in wage inequality since the 1960s. In addition to the long-run increase in wage differentials across workers with different levels of education and experience, inequality within narrowly defined groups (e.g., by race, education, and age/experience), called "residual" wage inequality, also rose dramatically. Despite its economic importance, the underlying cause of the latter trend is not fully understood.

In this paper, we show how panel data on wages can be used to separately identify factors commonly thought to explain the evolution of residual inequality: the evolution of (i) returns to unobserved skills, (ii) distributions of unobserved skills and skill growth rates, and (iii) the volatility of transitory shocks unrelated to skills. Building on the literature on earnings dynamics, our approach utilizes the long-run autocovariances in wage residuals as key sources of identification, so there is no need to observe anything about what workers do in their jobs, enabling our approach in most widely available panel data sets.

We estimate the evolution of returns to unobserved skills, variances of unobserved skills, and variances of the non-skill component of wages from 1970 to 2012 using data on log hourly wages for American men aged 16–64 years in the Panel Study of Income Dynamics. Our main finding is that the returns to unobserved skills fell considerably from the mid-1980s through the 1990s. The decline in skill returns was more dramatic for non-college-educated workers, consistent with the recent literature on polarization. However, the estimated time patterns for returns to unobserved skill are fundamentally different from those estimated in previous studies assuming time-invariant unobserved skill distributions.

The flip side of declining returns is that the variance of unobserved skills rose substantially, driving the increase in the residual variance of log wages. The widening skill distribution is largely explained by an increase in the variance of life cycle skill growth over time, not in the variance of initial skill levels across cohorts.

To understand why the returns to skill have fallen since the mid-1980s, we develop an assignment model of the labor market. Combining labor shares with our estimated skill returns and skill distributions, we identify changes in production technology and decompose changes in skill returns into supply vs. demand effects. Our estimates suggest that the fall in demand for skill explains most of the decline in returns for non-college-educated men, while both supply and demand shifts are similarly important for college-educated men.

Several of our findings are broadly consistent with the growing polarization literature, while highlighting the challenges faced by the simplest skill-biased technical change hypothesis, which argues that a steady increase in demand for skills has driven the increase in inequality. Further work is required to understand why skill demand declined during the 1990s, precisely when technological progress appears to have accelerated. Our results also suggest that more attention should be devoted to understanding the dramatic increase in unobserved skill inequality.

1 Introduction

The U.S. has experienced substantial and sustained growth in wage inequality since the 1960s. In addition to the long-run increase in wage differentials across workers with different levels of education and experience, inequality within narrowly defined groups (e.g., by race, education, and age/experience) also rose dramatically (see, e.g., Katz and Autor, 1999; Autor, Katz, and Kearney, 2008). While the interpretation of the latter trend is not fully understood, its underlying cause is economically important. Whether it reflects an increase in the returns to unobserved skills, variance of unobserved skills, or variance of short-term volatility unrelated to skills (or measurement error) is critical to our understanding of both the causes and welfare consequences of rising inequality.[1]

Since the seminal work of Juhn, Murphy, and Pierce (1993), many economists (implicitly or explicitly) equate the rising within-group, or residual, inequality with an increase in the returns to unobserved ability or skill (see, e.g., Card and Lemieux, 1996; Katz and Autor, 1999; Acemoglu, 2002; Autor, Katz, and Kearney, 2008). Indeed, this interpretation, along with the rising returns to observable skill, motivated an enormous and still influential literature on skill-biased technical change (SBTC).[2]

Challenging the conventional view, Lemieux (2006) demonstrates that the rise in residual inequality is at least partially explained by an increase in the variance of unmeasured skills resulting from composition changes in the labor market, especially in the late 1980s and 1990s, as the workforce shifted increasingly to older and more educated workers who exhibit greater within-group inequality. Lemieux (2006) and Gottschalk and Moffitt (2009) further argue that increasing measurement error and short-term volatility in wages may have also contributed to rising residual inequality.

A few studies have turned to richer data to incorporate additional measures of skill or occupational tasks, directly estimating their effects on wages at different points in time. Using the 1979 and 1997 Cohorts of the National Longitudinal Surveys of Youth (NLSY), Castex and Dechter (2014) estimate that the wage returns to cognitive achievement, as measured by the Armed Forces Qualifying Test (AFQT), declined substantially between the 1980s and early 2000s. By contrast, Deming (forthcoming) estimates that the returns to social skills have risen across these two cohorts. Among others, Autor, Levy, and Murnane (2003) and Autor and Dorn (2013) document a decline in demand for middle-skill workers

[1]Economists have long recognized the importance of distinguishing between transitory and permanent income shocks for understanding inequality in consumption and welfare. Several recent studies (e.g., Krueger and Perri, 2006; Blundell, Pistaferri, and Preston, 2008; Heathcote, Storesletten, and Violante, 2014) further show that conclusions about the nature and degree of consumption insurance (over time) depend critically on estimated correlations between consumption and income changes as well as the relative importance of transitory vs. permanent income shocks (over time).

[2]Many of these studies aimed specifically to explain rising residual inequality and returns to unobserved ability/skill (e.g., Galor and Tsiddon, 1997; Acemoglu, 1999; Caselli, 1999; Galor and Moav, 2000; Violante, 2002). Card and DiNardo (2002) question the influence of SBTC on the overall wage structure based largely on the failure of SBTC to explain the evolution of important between-group differences (largely across race and gender) in wages.

caused by the automation of routine tasks, which has led to a fall in the wages for workers in many middle-skill relative to low- and high-skill occupations, dubbed "polarization". Caines, Hoffmann, and Kambourov (2017) instead argue that occupational task complexity has become a stronger determinant of wages in recent years, more so than routineness.

While efforts to better measure skills and job tasks have greatly enriched our understanding of wage inequality, much of the cross-sectional variation in wages remains unexplained in these studies. More importantly, difficult measurement challenges have led to strong (often implicit) assumptions on the evolution of skills over the life cycle and across time. For example, Castex and Dechter (2014) and Deming (forthcoming) examine the effects of pre-market skills on wages, ignoring subsequent life cycle skill accumulation that may vary across workers and over time. Because the vast majority of studies taking a task-based approach do not use individual-level data on skills or job tasks, they implicitly assume that worker skills and tasks within each occupation are time invariant and attribute all time variation in wages across occupations to changes in the returns to skills/tasks.[3]

The literature studying the evolution of residual wage inequality relies largely on repeated samples of cross-sectional data, which makes it difficult to sort out changes in skill returns vs. the distributions of skills over time. Panel data are naturally more useful. Intuitively, if heterogeneity in skills is important, then workers earning a high wage today should continue, on average, to earn a high wage far into the future (even after the influence of transitory shocks has faded). As such, heterogeneity in unobserved skills implies that differences in wage residuals across workers should be predictive of long-term future residual differences, and residual autocovariances should not disappear for observations far apart in time. Furthermore, if unobserved skills become more important in the labor market (i.e., their returns increase) over time, then we should observe growing differences in predicted wage residuals conditional on initial residual differences.

We show how panel data on wages can be used to separately identify the evolution of (i) returns to unobserved skills, (ii) distributions of unobserved skills and skill growth rates, and (iii) the volatility of transitory shocks unrelated to skills. Building on the literature on earnings dynamics, our key source of identification (long-run autocovariances in wage residuals) motivates a simple instrumental variable strategy for estimating the returns to skill over time, even when life cycle skill growth varies systematically across individuals and is subject to time-varying idiosyncratic shocks.[4] This approach also allows for a

[3]Using data with individual-level measures of job tasks, Spitz-Oener (2006) shows that most task changes in Germany over the 1980s and 1990s actually occurred within occupations. Autor and Handel (2013) document substantial heterogeneity in worker tasks within occupations that are powerful predictors of wage variation (within occupational, demographic, and education groups). Their finding that person-level job tasks vary systematically across demographic groups within narrowly defined occupations suggests that worker skills and job tasks are unlikely to be time invariant within occupations as the composition of the workforce changes.

[4]The voluminous literature on earnings dynamics focuses on a different question from ours: identifying the changing importance of permanent vs. transitory shocks in earnings and the resulting implications for consumption and wealth inequality

very general structure for transitory non-skill shocks. Once the returns to skill have been estimated, it is straightforward to estimate the distribution of unobserved skills, skill growth, and non-skill shocks over time. Importantly, there is no need to observe anything about what workers do on their jobs, enabling our approach in most widely available panel data sets.

We estimate the evolution of returns to unobserved skills, variances of unobserved skills, and variances of the non-skill component of wages from 1970 to 2012 using data on log hourly wages for men ages 16–64 in the Panel Study of Income Dynamics (PSID), performing separate analyses by college attendance. Our main finding is that the returns to unobserved skills were relatively stable from 1970 to the mid-1980s, then *fell* considerably through the 1990s, stabilizing thereafter. The drop in estimated returns reflects a sharp decline in the predictability of long-term wage residual differences conditional on earlier differences and is robust to different estimation strategies and to very general specifications regarding heterogeneity across cohorts and experience levels. The decline in skill returns was more dramatic for non-college-educated workers, consistent with the recent literature on polarization (Autor, Levy, and Murnane, 2003; Autor, Katz, and Kearney, 2008; Acemoglu and Autor, 2011; Autor and Dorn, 2013).

The flip side of declining returns is that the variance of unobserved skills rose substantially, driving the increase in the residual variance of log wages. The widening skill distribution is largely explained by an increase in the variance of life cycle skill growth over time, not in the variance of initial skill levels across cohorts.[5] We further show that the increasing variance of skill growth reflects increases in the variances of both idiosyncratic skill growth shocks (consistent with the notion of growing economic turbulence studied by Ljungqvist and Sargent (1998)) and heterogeneous systematic life cycle skill growth.[6]

Our results highlight the importance of accounting for changes in the distribution of unobserved skills over time. Our estimated time patterns for returns to unobserved skill are fundamentally different from those estimated in previous studies assuming time-invariant unobserved skill distributions (e.g., Juhn, Murphy, and Pierce, 1993; Moffitt and Gottschalk, 2012). They are more consistent with the falling returns to AFQT between the 1980s and early 2000s as estimated by Castex and Dechter (2014).

To help interpret our empirical findings, we develop a simple demand and supply framework based on the assignment model of Sattinger (1979). In this model, the returns to skills are determined by the assignment of workers with heterogeneous skills to jobs with heterogeneous productivity (e.g., different tasks or capital quality), as well as the technology of production itself. More skilled workers earn more

(e.g., Gottschalk and Moffitt, 1994; Blundell and Preston, 1998; Haider, 2001; Moffitt and Gottschalk, 2002; Meghir and Pistaferri, 2004; Bonhomme and Robin, 2010; Heathcote, Storesletten, and Violante, 2010; Heathcote, Perri, and Violante, 2010; Moffitt and Gottschalk, 2012; Blundell, Graber, and Mogstad, 2015).

[5]Since we estimate widening skill growth distributions within education and experience groups, these trends are not accounted for in the composition adjustments of Lemieux (2006).

[6]Baker and Solon (2003), Guvenen (2009), and Moffitt and Gottschalk (2012) estimate heterogeneity in skill/income growth profiles but do not estimate changes in the extent of that heterogeneity over time.

partly because they work at more productive jobs. As a result, the skill distribution affects the return to skill by changing the equilibrium assignment. An increase in the variance of skills reduces the returns to skills by shrinking the productivity differential across jobs among workers of different skill levels. We recover the demand and supply factors by combining our estimates of the returns to skills and variance of skills with key restrictions implied by the model. Our estimates suggest that both demand and supply factors played important roles in the declining returns to unobserved skills since the mid-1980s for both college-educated and non-college-educated workers. The decline in skill demand was a more important factor than supply shifts for non-college-educated workers, consistent with the automation of routine tasks in middle-skill jobs, as emphasized by Autor, Levy, and Murnane (2003), Autor and Dorn (2013), and many others.

This paper proceeds as follows. In Section 2, we provide identification results for our model where the returns to unobserved skills, the variance of unobserved skills, and the variance of the non-skill component of earnings change over time. Section 3 describes the PSID data used in our empirical analysis and highlights key features of the data relevant to the evolution of skill returns. Sections 4 and 5 report our empirical findings. Section 6 develops our assignment model of the labor market and uses it to interpret the evolution of estimated returns to unobserved skills in terms of changes in demand and supply. Section 7 concludes.

2 Identifying the Returns to and Distributions of Unobserved Skills

In this section, we describe a general specification for wages that is consistent with much of the empirical literature on residual wage inequality and our theoretical framework in Section 6. We then establish conditions under which the time series for the returns to unobserved skills, the variances of unobserved skills, and the variances of transitory non-skill shocks (or measurement error) are identified.

2.1 Log Wage Functions

We consider the following specification for log wages motivated by the literature on unobserved skills (e.g., Juhn, Murphy, and Pierce, 1993; Card and Lemieux, 1994; Chay and Lee, 2000; Lemieux, 2006):

$$\ln W_{i,t} = f_t(\boldsymbol{x}_{i,t}) + \underbrace{\mu_t \theta_{i,t} + \varepsilon_{i,t}}_{\equiv w_{i,t}}, \tag{1}$$

where $W_{i,t}$ reflects wages, $\boldsymbol{x}_{i,t}$ observed characteristics (e.g., education, race, experience), and $\theta_{i,t}$ (log) unobserved skill for individual i in period t.[7] The time-varying function $f_t(\cdot)$ incorporates effects of observable characteristics on period t log wages, while the "residual" $w_{i,t} \equiv \mu_t \theta_{i,t} + \varepsilon_{i,t}$ reflects the contributions of unobserved skills and idiosyncratic non-skill shocks $\varepsilon_{i,t}$ (including measurement error).[8] In Section 6, we develop and study an assignment model of the labor market that produces log wage functions of this form.[9]

The log wage residual $w_{i,t}$ is the primary focus of our efforts to identify and estimate the evolution of "returns" to unobserved skill, distributions of unobserved skill, and volatility of non-skill shocks.

2.2 Identification

Suppose that we observe log wage residuals from equation (1) for a large number of individuals $i = 1, 2, \ldots, N$ for periods $t = \underline{t}, \underline{t}+1, \ldots, \bar{t}$:

$$w_{i,t} = \mu_t \theta_{i,t} + \varepsilon_{i,t}, \tag{2}$$

where $\theta_{i,t}$ represents unobserved (log) skill, μ_t the period t return to unobserved skill, and $\varepsilon_{i,t}$ idiosyncratic shocks. Since $w_{i,t}$ is a mean zero residual, we normalize $\theta_{i,t}$ and $\varepsilon_{i,t}$ so that both are mean zero for all t. This implies that unobserved skill growth innovations,

$$\nu_{i,t} \equiv \theta_{i,t} - \theta_{i,t-1}$$

are also mean zero for all t.[10] We further assume that $\text{Cov}(\theta_t, \varepsilon_{t'}) = \text{Cov}(\nu_t, \varepsilon_{t'}) = 0$ for all t, t'.[11] Thus, any shocks related to skills are embedded in $\theta_{i,t}$, while $\varepsilon_{i,t}$ reflects factors unrelated to skills. We note that individuals may come from different cohorts (i.e., different years of labor market entry), which we discuss further below.

[7] The assumption of separability between $\boldsymbol{x}_{i,t}$ and $\theta_{i,t}$ is both common and convenient, although not necessary. One could simply condition the analysis that follows on $\boldsymbol{x}_{i,t}$. Indeed, much of our empirical analysis separately studies non-college-educated and college-educated workers.

[8] Chay and Lee (2000), Card and Lemieux (1994), and Lemieux (2006) consider the same log wage residual decomposition; however, they assume that the variances of skills within observable groups (e.g., education, experience, race) are time invariant. Thus, their approaches account only for changes in the overall variance of unobserved skills due to changes in the composition of workers across observable types. In estimating the importance of these composition changes, Lemieux (2006) further ignores any variation in the transitory component, $\varepsilon_{i,t}$. Our use of panel data enables a much more general analysis.

[9] Notice that wage levels are non-linear in unobserved skill. As such, variation in μ_t over time is inconsistent with perfect substitutability across workers of different skill levels, since perfect substitutability would imply log wages functions that are additively separable in "prices" and skills.

[10] Average skill growth rates, which may vary by observable characteristics, are reflected in changes in $f_t(\boldsymbol{x}_{i,t})$.

[11] Let x_t be a random variable and its realization for individual i be $x_{i,t}$. Denote its cross-sectional second moments by $\text{Var}(x_t)$ and $\text{Cov}(x_t, x_{t'})$.

Central to our approach is the classical idea of Friedman and Kuznets (1954) that earnings consist of a permanent component related to skills and a transitory component that reflects short-run variation unrelated to skills. Although the transitory component, which may include measurement error, can be serially correlated, the correlation between transitory components far apart in time is likely to be quite small. Taking this to the limit, we assume that there exists some $k > 0$ such that $\text{Cov}(\varepsilon_t, \varepsilon_{t'}) = 0$ for $|t' - t| \geq k$, so the "long" autocovariance of wage residuals reflects only the skill-related component (Carroll, 1992; Moffitt and Gottschalk, 2012):

$$\text{Cov}(w_t, w_{t'}) = \mu_t \mu_{t'} \text{Cov}(\theta_t, \theta_{t'}), \quad \text{for } |t' - t| \geq k. \tag{3}$$

In short, we assume that persistent long-term differences in wages across workers are driven by lasting differences in skills. Appendix A.1 shows that our main identification results continue to hold when the "transitory" component $\varepsilon_{i,t}$ contains an autoregressive component, such that the serial correlation in non-skill shocks depreciates exponentially over time but never fully disappears. Empirical results are also quite similar in this case.

The properties of skill growth innovations, $\nu_{i,t}$, are important for identification. We begin by assuming that these innovations are serially uncorrelated, then show how systematic heterogeneity in skill growth rates (over most of the life cycle) can be incorporated.

2.2.1 Serially Uncorrelated Skill Growth Shocks

First, consider the case in which unobserved skill growth innovations $\nu_{i,t}$ are uncorrelated with past skill levels and growth shocks; i.e., $\text{Cov}(\theta_{t'}, \nu_t) = 0$ for all $t' < t$ and $\text{Cov}(\nu_t, \nu_{t'}) = 0$ for all $t \neq t'$. This implies that skill levels are persistent in the sense that, for all $t' \leq t$, $\text{Cov}(\theta_t, \theta_{t'}) = \text{Var}(\theta_{t'})$. Together with equation (3), this implies that $\text{Cov}(w_t, w_{t'}) = \mu_t \mu_{t'} \text{Var}(\theta_{t'})$ for $t' \leq t - k$, so the following ratio of residual covariances identifies the ratio of skill returns:

$$\frac{\text{Cov}(w_t, w_{t'})}{\text{Cov}(w_{t-1}, w_{t'})} = \frac{\mu_t \mu_{t'} \text{Var}(\theta_{t'})}{\mu_{t-1} \mu_{t'} \text{Var}(\theta_{t'})} = \frac{\mu_t}{\mu_{t-1}}, \quad \text{for } t' < t - k. \tag{4}$$

This suggests that μ_t / μ_{t-1} can be easily estimated by regressing $w_{i,t}$ on $w_{i,t-1}$ using sufficiently lagged $w_{i,t'}$ as an instrumental variable (IV).

An IV estimation approach can also be motivated using the framework of Holtz-Eakin, Newey, and Rosen (1988). We can substitute in for $\theta_{i,t}$ in equation (2) to obtain an expression for $w_{i,t}$ in terms of $w_{i,t-1}$:

$$w_{i,t} = \mu_t \left(\frac{w_{i,t-1} - \varepsilon_{i,t-1}}{\mu_{t-1}} + \nu_{i,t} \right) + \varepsilon_{i,t} = \frac{\mu_t}{\mu_{t-1}} w_{i,t-1} + \left(\varepsilon_{i,t} - \frac{\mu_t}{\mu_{t-1}} \varepsilon_{i,t-1} + \mu_t \nu_{i,t} \right). \tag{5}$$

suggesting that lagged residuals $w_{i,t-1}$ might serve as a proxy for unobserved skills. However, because $w_{i,t-1} = \mu_{t-1}\theta_{i,t-1} + \varepsilon_{i,t-1}$ is a "noisy" measure of unobserved skill $\theta_{i,t}$, it is correlated with the measurement error $\varepsilon_{i,t-1}$ (and $\varepsilon_{i,t}$ if $\text{Cov}(\varepsilon_t, \varepsilon_{t-1}) \neq 0$). Simply regressing $w_{i,t}$ on $w_{i,t-1}$ would, therefore, produce a biased estimate of μ_t/μ_{t-1}. To address this problem, wage residuals from the distant past (i.e., any $w_{i,t'}$ for $t' < t-k$) can be used as instrumental variables, since they are correlated with $w_{i,t-1}$ (through unobserved skills) but uncorrelated with $\varepsilon_{i,t-1}$, $\varepsilon_{i,t}$, and $\nu_{i,t}$. Therefore, μ_t/μ_{t-1} can be estimated using IV methods for all but the first few sample years; i.e., $t > \underline{t} + k$.

Future wage residuals are not valid instruments in equation (5), because skill growth has permanent effects on future skills. Indeed, IV regression using future wage residuals as instruments produces an upward biased estimate of μ_t/μ_{t-1} with the bias proportional to the variance of skill growth:

$$\frac{\text{Cov}(w_t, w_{t''})}{\text{Cov}(w_{t-1}, w_{t''})} = \frac{\mu_t\mu_{t''}\text{Var}(\theta_t)}{\mu_{t-1}\mu_{t''}\text{Var}(\theta_{t-1})} = \frac{\mu_t}{\mu_{t-1}}\left[1 + \frac{\text{Var}(\nu_t)}{\text{Var}(\theta_{t-1})}\right] \quad \text{for } t'' \geq t+k. \tag{6}$$

Since IV estimates using past residuals consistently estimate μ_t/μ_{t-1}, the ratio of IV estimates using future vs. past residuals as instruments can be used to identify the importance of skill growth shocks (relative to variation in lagged skill levels):

$$\frac{\text{Var}(\nu_t)}{\text{Var}(\theta_{t-1})} = \frac{\text{Cov}(w_t, w_{t''})/\text{Cov}(w_{t-1}, w_{t''})}{\text{Cov}(w_t, w_{t'})/\text{Cov}(w_{t-1}, w_{t'})} - 1, \quad \text{for } t' + k < t \leq t'' - k.$$

The bias for μ_t/μ_{t-1} when using future residuals as instruments poses an identification challenge for μ_t in early sample periods when past observations are not available to serve as instruments. Fortunately, two additional conditions enable estimation of μ_t/μ_{t-1} for early years by differencing out this bias across cohorts. To see this, let c reflect the period of labor market entry ("cohort"), and suppose that two cohorts exist, c and $\tilde{c}$, such that $\text{Var}(\theta_{t-1}|c) \neq \text{Var}(\theta_{t-1}|\tilde{c})$ and $\text{Var}(\nu_t|c) = \text{Var}(\nu_t|\tilde{c})$.[12] The first condition is likely to hold quite generally. For example, differences in the variance of initial skill levels would contribute to different variances later in life. Even if initial skill distributions were identical across cohorts, the older cohort is likely to have accumulated more skill growth innovations over its longer career. The second condition holds when the skill growth variance depends only on time (and not experience) or when there is a non-monotonic experience trend in the variance of skill changes. For example, young workers may experience greater variation in skill growth than middle-aged workers due to differences in training or learning opportunities, while older workers may have a greater variance in skill changes due to differences in health shocks or skill obsolescence. Indeed, Baker and Solon (2003) and Blundell, Graber, and Mogstad (2015) estimate a U-shaped age profile for the variance of earnings shocks. In this case, the ratio of the difference of the long autocovariances between cohorts identifies the ratio of skill returns for

[12] We assume that all previous assumptions on the covariance structure of θ_t, ε_t, and ν_t hold conditional on c.

early periods:

$$\frac{\text{Cov}(w_t, w_{t'}|c) - \text{Cov}(w_t, w_{t'}|\tilde{c})}{\text{Cov}(w_{t-1}, w_{t'}|c) - \text{Cov}(w_{t-1}, w_{t'}|\tilde{c})} = \frac{\mu_t \mu_{t'} \left[\text{Var}(\theta_{t-1}|c) - \text{Var}(\theta_{t-1}|\tilde{c}) \right]}{\mu_{t-1} \mu_{t'} \left[\text{Var}(\theta_{t-1}|c) - \text{Var}(\theta_{t-1}|\tilde{c}) \right]} = \frac{\mu_t}{\mu_{t-1}}, \quad \text{for } t' \geq t + k. \quad (7)$$

With this, we can identify the sequence of all μ_t over the sample period, where we must normalize one skill return in a single time period: say, t^*. Normalizing $\mu_{t^*} = 1$ effectively sets the units for unobserved skill in terms of dollars per hour of work in year t^*.

Once the sequence of all μ_t has been identified, the variances of unobserved skills for all $t \leq \bar{t} - k$ are identified from the covariance between current and future wage residuals $\text{Var}(\theta_t) = \text{Cov}(w_t, w_{t'})/(\mu_t \mu_{t'})$ for $t' \geq t + k$. Although the return to unobserved skill is identified for all periods, the variance of unobserved skills is not identified for later periods $t > \bar{t} - k$ (without further assumptions), because it is impossible to distinguish between unobserved skills and wage shocks without observing future wage residuals. Having identified the variance of unobserved skills over time, it is straightforward to then identify $\text{Var}(v_t) = \text{Var}(\theta_t) - \text{Var}(\theta_{t-1})$ and $\text{Var}(\varepsilon_t) = \text{Var}(w_t) - \mu_t^2 \text{Var}(\theta_t)$.

We summarize the above discussion in Proposition 1, where we also note that the variances of unobserved skills, skill growth shocks, and non-skill transitory shocks can be identified separately by cohort.

Proposition 1. *Assume: (i) there exists $k > 0$ such that $\bar{t} - \underline{t} \geq 2k$ and $\text{Cov}(\varepsilon_t, \varepsilon_{t'}|c) = 0$ for all (c, t, t') such that $|t' - t| \geq k$, (ii) $\text{Cov}(\varepsilon_t, \theta_{t'}|c) = \text{Cov}(\varepsilon_t, v_{t'}|c) = 0$ for all (c, t, t'), (iii) $\text{Cov}(v_t, \theta_{t-j}|c) = 0$ for all (c, t), and $j > 0$, and (iv) $\text{Var}(\theta_{t-1}|c) \neq \text{Var}(\theta_{t-1}|\tilde{c})$ and $\text{Var}(v_t|c) = \text{Var}(v_t|\tilde{c})$ for some $c \neq \tilde{c}$. Then, (i) μ_t is identified for all t up to a normalization $\mu_{t^*} = 1$ for some period t^*, (ii) $\text{Var}(\theta_t|c)$ and $\text{Var}(\varepsilon_t|c)$ are identified for all (c, t) such that the cohort c is observed both in period t and some other period $t' \geq t + k$, and (iii) $\text{Var}(v_t|c)$ is identified for all (c, t) such that $\text{Var}(\theta_t|c)$ and $\text{Var}(\theta_{t-1}|c)$ are identified.*

Our identification strategy has relied on the assumption that non-skill shocks, $\varepsilon_{i,t}$, become serially uncorrelated when observations are far enough apart. This is not critical, although identification is most transparent in this case. Appendix A.1 provides an analogous identification analysis when $\varepsilon_{i,t}$ follows an ARMA(1,q) process, such that the serial correlation in non-skill shocks never fully disappears.

2.2.2 Heterogeneity in Life Cycle Skill Growth

We now consider the possibility that unobserved skill growth innovations $v_{i,t}$ may be correlated over time as in the heterogeneous income profile (HIP) models estimated in, for example, Haider (2001), Baker and Solon (2003), Guvenen (2009), and Moffitt and Gottschalk (2012). We consider a flexible process

governing this skill growth heterogeneity, assuming

$$v_{i,t} = \tau_t(c_i)\delta_i + \tilde{v}_{i,t}. \tag{8}$$

where δ_i is a mean zero individual-specific life cycle growth rate factor and the $\tau_t(c) \geq 0$ terms allow for variation in systematic skill growth across time and cohorts/experience. If we let ψ_i reflect the initial skill level for an individual just entering the labor market, then the level of unobserved skill for individual i from cohort c_i in year t can be written as

$$\theta_{i,t} = \psi_i + \sum_{j=0}^{t-c_i-1} \tau_{t-j}(c_i)\delta_i + \sum_{j=0}^{t-c_i-1} \tilde{v}_{i,t-j}.$$

We assume that idiosyncratic skill growth shocks $\tilde{v}_{i,t}$ are serially uncorrelated and uncorrelated with initial skills ψ_i and systematic skill growth δ_i; however, we make no assumptions about the correlation between heterogeneous skill growth rates δ_i and initial skill levels ψ_i. We continue to assume that non-skill shocks $\varepsilon_{i,t}$ are uncorrelated with all skill-related components ψ_i, δ_i, and $\tilde{v}_{i,t'}$ for all t, t'. Altogether, these assumptions imply the following variance of skills:

$$\mathrm{Var}(\theta_t|c) = \mathrm{Var}(\psi|c) + \left(\sum_{j=0}^{t-c-1} \tau_{t-j}(c)\right)^2 \mathrm{Var}(\delta|c) + 2\sum_{j=0}^{t-c-1} \tau_{t-j}(c)\,\mathrm{Cov}(\psi, \delta|c) + \sum_{j=0}^{t-c-1} \mathrm{Var}(\tilde{v}_{t-j}|c). \tag{9}$$

This includes two new terms reflecting (i) the variance of accumulated (systematic) skill growth and (ii) the covariance between this accumulated skill growth and initial skills. The covariance between skills in periods t and $t' < t$ can be written as

$$\mathrm{Cov}(\theta_t, \theta_{t'}|c) = \mathrm{Var}(\theta_{t'}|c) + \sum_{j=t'+1}^{t} \tau_j(c)\,\mathrm{Cov}(\theta_{t'}, \delta|c). \tag{10}$$

Unless $\sum_{j=t'+1}^{t} \tau_j(c) = 0$, it is clear that $\mathrm{Cov}(\theta_t, \theta_{t'}|c)$ will not generally equal $\mathrm{Var}(\theta_{t'}|c)$, and the IV approach described in the previous subsection (see equation (4)) cannot be used to identify/estimate μ_t/μ_{t-1}.[13] An additional assumption is needed.

Human capital theory (Becker, 1964; Ben-Porath, 1967) predicts that skill investment and accumulation should be negligible as workers approach the end of their careers, a prediction confirmed by the lack of wage growth among most older workers. While assuming zero skill growth among older workers

[13]Notice that $\mathrm{Cov}(\theta_{t'}, \delta|c) = \left(\sum_{j=0}^{t-c-1} \tau_{t-j}(c)\right) \mathrm{Var}(\delta|c) + \mathrm{Cov}(\psi, \delta|c)$, which can be negative very early in workers' careers if initial skill levels and skill growth rates are negatively correlated.

would enable identification of μ_t/μ_{t-1}, such an assumption is stronger than needed. Instead, it is sufficient to assume that there is no unobserved heterogeneity in skill growth among the most experienced workers; i.e., $\tau_t(c) = 0$ for all (t, c) satisfying $e = t - c \geq \bar{e}$.[14] This assumption implies that skill innovations are serially uncorrelated and that $\text{Cov}(\theta_t, \theta_{t'}|c) = \text{Var}(\theta_{t'}|c)$ for workers with $e \geq \bar{e}$. Based on the analysis of Section 2.2.1, the returns to skill can be identified and estimated by following these workers over time.[15] No other assumptions are needed regarding the structure of skill growth heterogeneity across time or cohort/experience (i.e., $\tau_t(c)$) over the rest of the life cycle. In practice, many cohorts may be needed to recover a long time series of skill returns μ_t by using overlapping subsamples of sufficiently experienced workers. See Appendix A.2 for details on identification of the full model.

2.3 Returns to Skill and Predicted Future Wage Differences

The evolution of returns to skill are directly related to predicted differences in wages across workers given any prior differences. Assuming skill and non-skill shocks are mean independent of past skill levels (i.e., $\text{E}[\nu_t|\theta_{t'}] = \text{E}[\varepsilon_t|\theta_{t'}] = 0$ for all $t > t'$) implies that $\text{E}[w_t|w_{t'}] = \mu_t \left(w_{t'} - \text{E}[\varepsilon_{t'}|w_{t'}]\right)/\mu_{t'}$ for $t' \leq t - k$.[16] Thus, for any given year t' differences in residuals across workers, long-term differences in expected future residuals, $\text{E}[w_t|w_{t'}]$, will increase (decrease) over time as the returns to skill μ_t increase (decrease):

$$\text{E}\left[w_t|w_{t'} = w^H\right] - \text{E}\left[w_t|w_{t'} = w^L\right] = \mu_t \left(\frac{w^H - w^L + \text{E}[\varepsilon_{t'}|w_{t'} = w^L] - \text{E}[\varepsilon_{t'}|w_{t'} = w^H]}{\mu_{t'}}\right) \tag{11}$$

for $t' \leq t - k$. Intuitively, wage differences are related to skill differences when there are positive returns to skill. As a result, workers with a high wage today will also tend to have a high wage in the future, even after the influence of any transitory non-skill shocks have disappeared. If the returns to skill increase over time, then differences in expected wages for any two individuals should also grow in the long run.

[14] This assumption is weaker than the assumption of zero skill growth for older workers used to identify skill price patterns in Heckman, Lochner, and Taber (1998) and Bowlus and Robinson (2012) and the distribution of human capital shocks in Huggett, Ventura, and Yaron (2011). Our assumption accommodates any average level of skill growth, as well as systematic differences in skill growth based on observable characteristics like education, experience, and time.

[15] As discussed in Appendix A.2, the experience level $\bar{e}$ above which $\tau_t(c) = 0$ must leave enough years prior to retirement to identify skill returns based on workers with $e \geq \bar{e}$. For example, μ_t/μ_{t-1} for $t > \underline{t} + k$ is identified if $\bar{e} + k$ is less than the maximum experience level used in the analysis.

[16] In the special case where $\varepsilon_{t'}$ and $\theta_{t'}$ are both normally distributed, this expression simplifies nicely to $\text{E}[w_t|w_{t'}] = \frac{\mu_t}{\mu_{t'}}\left[1 - \frac{\text{Var}(\varepsilon_{t'})}{\text{Var}(w_{t'})}\right] w_{t'}$.

3 PSID Data

To estimate the evolution of returns to unobserved skills, variances of unobserved skills and skill growth innovations, and variances of transitory non-skill shocks (including measurement error), we utilize data for American men from the PSID. The PSID is a longitudinal survey of a representative sample of individuals and families in the U.S. beginning in 1968. The survey was conducted annually through 1997 and biennially since. We use data collected from 1971 through 2013. Since earnings were collected for the year prior to each survey, our analysis studies hourly wages from 1970 to 2012. (Key findings also hold for annual earnings.)[17]

Our sample is restricted to male heads of households from the core (SRC) sample.[18] We use earnings (total wage and salary earnings, excluding farm and business income) from any year these men were ages 16–64, had potential experience of 1–40 years, had positive wage and salary income, had positive hours worked, and were not enrolled as a student. We calculate the wage measure we use in our analysis by dividing annual earnings by annual hours worked, trimming the top and bottom 1% of all wages within year by college- vs. non-college-educated status and 10-year experience cells. The resulting sample contains 3,766 men and 44,547 person-year observations – roughly 12 observations for each individual.

Our sample is composed of 92% white, 6% black, and 1% Hispanic men with an average age of 39 years. We create seven education categories based on current years of completed schooling: 1–5 years, 6–8 years, 9–11 years, 12 years, 13–15 years, 16 years, and 17 or more years. College-educated workers are defined as those with more than 12 years of schooling. In our sample, 13% of respondents finished fewer than 12 years of schooling, 35% had exactly 12 years of completed schooling, 21% completed some college (13–15 years), 21% completed college (16 years), and 10% had more than 16 years of schooling.

Our analysis focuses on log wage residuals $w_{i,t}$ from equation (1) after controlling for differences in educational attainment, race, and experience. Specifically, we estimate $f_t(\boldsymbol{x}_{i,t})$ by year and college- vs. non-college-educated status from separate linear regressions of log hourly wages on indicators for each year of potential experience, race, and our educational attainment categories, along with interactions between race and education indicators and a third-order polynomial in experience.

Figure 1 shows the total variance, between-group variance, and within-group variance (variance of residuals) of log wages over time. The variance of log wages increases sharply in the early 1980s and after the late 1990s. The evolution of the within-group variance closely mirrors this pattern. The within-group variance explains a larger share of the total variance than the between-group variance, and it also explains

[17] Results available upon request. Also, see Lochner and Shin (2014) for similar results using a slightly different specification.

[18] We exclude those from any PSID oversamples (SEO, Latino) as well as those with zero individual weights. The earnings questions we use are asked only of household heads. We also restrict our sample to those who were heads of household and not students during the survey year of the observation of interest as well as two years earlier. Our sampling scheme is very similar to that of Moffitt and Gottschalk (2012).

an increasing share of the total variance since the early 1990s.

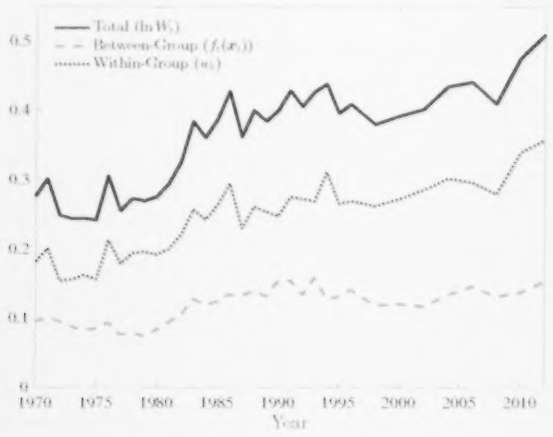


Figure 1: Between- and Within-Group Variances of Log Wages

Figure 2 also shows the widening of the residual distribution over time, reporting average log wage residuals within each quartile. Consistent with Figure 1, the distribution widened most during the early 1980s and then again after 2000.

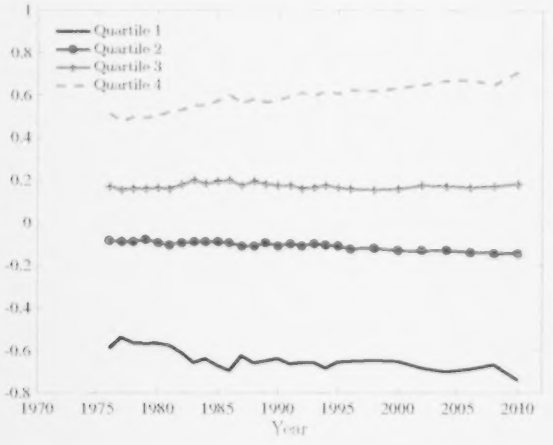


Figure 2: Average Log Wage Residuals by Quartile

As discussed in Section 2.3, the evolution of expected future wage residuals conditional on some "base" year residuals w_b, $\mathrm{E}[w_t|w_b]$, is informative about the evolution of skill returns. To explore this, we categorize workers based on their residual quartile in three different "base" years (1970, 1980, and 1990), then calculate their average residuals in subsequent years $t = b + 6, ..., b + 20$. These averages

are reported in Figure 3, which has three lines for each quartile associated with each of the three "base" years. Each line traces out estimates of $\mathrm{E}\left[w_t | w_b \in Q_b^j\right]$ for different t years where Q_b^j reflects quartile j in "base" year b. Not surprisingly, the lines are always higher for workers from higher initial residual quartiles. Consistent with an important role for unobserved skills, those earning higher wages in any given base year also earn more, on average, up to 20 years later. More interestingly, the lines are fairly flat over the late 1970s and early 1980s, as well as in the 2000s; however, they sharply converge over the late 1980s and 1990s.[19] This suggests that the returns to skill *fell* over the middle years of our sample (see equation (11)), despite modest growth in residual inequality at the time (see Figures 1 and 2).

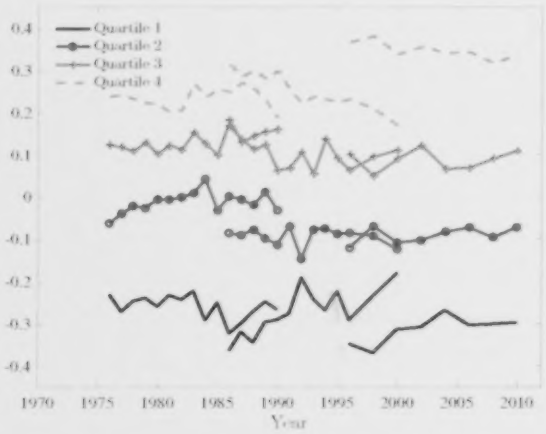


Figure 3: Average Predicted Log Wage Residuals by Baseline Residual Quartile

Long autocovariances in wage residuals play a central role in identifying changes in the returns to unobserved skill (see Section 2). Figure 4 reports $\mathrm{Cov}(w_b, w_t)$ for $t = b+6, ..., b+20$ with each line reporting autocovariances for a different "base" year b and 15 subsequent years.[20] For example, the left-most line beginning in 1976 reflects autocovariances for $b = 1970$ and values of t ranging from 1976–1990. If heterogeneity in systematic unobserved skill growth is negligible and $t - b$ is large enough such that transitory shocks are uncorrelated, then $\mathrm{Cov}(w_b, w_t) = \mu_t \mu_b \mathrm{Var}(\theta_b)$ and following each line over t is directly informative about the evolution of μ_t, while the shifts up or down across lines at a given date t are informative about differences in $\mu_b \mathrm{Var}(\theta_b)$. The sharply declining autocovariances over the late 1980s and 1990s (regardless of the base year) suggest that the returns to unobserved skill fell over that period. The time trends for autocovariances were much weaker during earlier and later years, consistent

[19]While not shown, this general pattern holds when using other "base" years as well. The differences in levels across lines for any given quartile reflect differences in the base years' wage distribution – see equation (11).

[20]Figure E-1 in Appendix E shows that sample attrition due to non-response or aging/retirement does not affect the autocovariance patterns documented here.

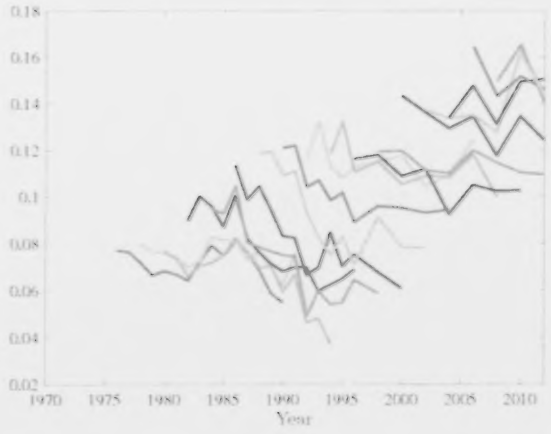


Figure 4: Autocovariances for Log Wage Residuals

with more stable returns. Finally, the upward shifting lines beginning in the 1980s, coupled with declining or constant skill returns, indicate an increasing variance of unobserved skills.

If heterogeneity in unobserved skill growth is important, then the residual covariances are more difficult to interpret, since $\text{Cov}(\theta_b, \theta_t)$ does not generally equal $\text{Var}(\theta_b)$ (see equation (10)). In this case, it is useful to focus on more experienced workers for whom differences in unobserved skill growth should be negligible. Figure 5 reveals very similar autocovariance patterns to Figure 4 when restricting the sample to men with 16-30 years of experience as of baseline b years.[21] Thus, our conclusions about the declining returns to skill and growing variance of unobserved skill do not appear to be affected by heterogeneity in systematic unobserved skill growth.

As emphasized by the literature on "polarization" in the U.S. labor market (Autor, Levy, and Murnane, 2003; Autor, Katz, and Kearney, 2008; Acemoglu and Autor, 2011; Autor and Dorn, 2013), wage inequality evolved differently for non-college- and college-educated workers. Figure 6 shows that the rise in residual inequality over the early 1980s was stronger among non-college-educated workers; however, residual inequality began to fall among non-college-educated workers in the mid-1980s while it continued to increase among college-educated workers. It is natural to ask whether these differential trends reflect differences in the evolution of returns to skill by educational attainment.

To examine this, Figure 7 reports autocovariances separately for non-college- and college-educated

[21]Figure E-2 in Appendix E shows qualitatively similar autocovariances for men with 1–15 years of experience (i.e., lines generally declining in t over the late 1980s and 1990s, while shifting upwards across base years b); however, the patterns are much more muted. The weaker declines in $\text{Cov}(w_b, w_t)$ with changes in t (among less experienced workers) are consistent with a positive correlation between baseline skills θ_b and individual-specific skill growth rates δ for young workers (see equation (10)). The lower levels and more modest upward shifts in the lines across base years b are consistent with less dispersion in skills among less experienced workers.

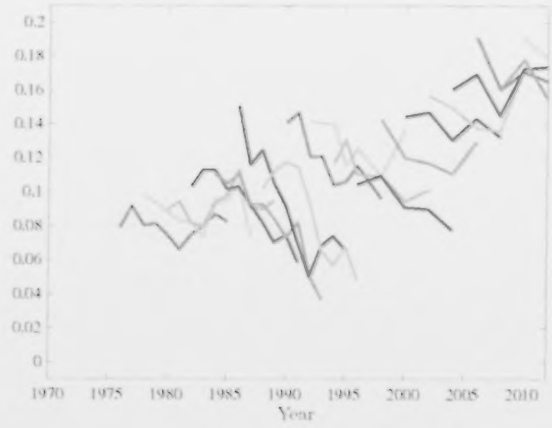


Figure 5: Autocovariances for Log Wage Residuals (16–30 Years of Experience)

men (analogous to those in Figure 4). The time patterns are qualitatively similar for both education groups, with two noteworthy differences: First, the autocovariance lines continue declining for non-college-educated men throughout the early 2000s when they flatten out for college-educated men. This suggests that the returns to skill continued falling for non-college-educated men several years after they stabilized for college-educated men. Second, the lines shift more strongly upward over the late 1990s and early 2000s for college-educated men, suggesting that the distribution of skills may have widened more for them over this period.[22]

We explore these autocovariance patterns and their implications for the returns to skill, variance of unobserved skill, and variance of transitory non-skill wage shocks more deeply in the empirical analysis presented in the next two sections.

4 Instrumental Variable Estimation of Skill Returns

In this section, we estimate growth rates in the returns to unobserved skill based on the IV strategy described in Section 2. Given our data are available only every other year later in the sample period, we modify the strategy slightly to estimate two-year growth rates, $(\mu_t - \mu_{t-2})/\mu_{t-2}$, using two-stage least squares (2SLS) with multiple lags of log wage residuals as instruments. Substituting in for $\theta_{i,t} = \frac{w_{i,t-2} - v_{i,t-2}}{\mu_{t-2}} + \nu_{i,t-1} + \nu_{i,t}$

[22]This is not necessarily the case, however, since the lines shift upward if μ_t Var(θ_t) increased. For non-college-educated men, μ_t is declining over the early 2000s (based on the slopes of all lines over that period), while it is not for college-educated men. Thus, we would expect smaller upward shifts in the lines for non-college-educated relative to college-educated men even with the same increases in skill variances.

Figure 6: Variance of Log Wage Residuals by Educational Attainment

in equation (2), subtracting $w_{i,t-2}$ from both sides, and rearranging yields

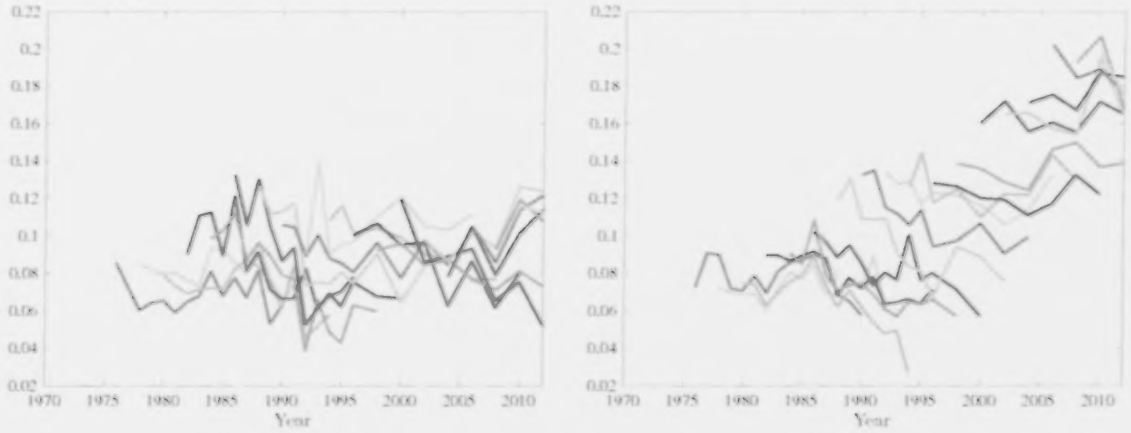
$$w_{i,t} - w_{i,t-2} = \left(\frac{\mu_t - \mu_{t-2}}{\mu_{t-2}}\right) w_{i,t-2} + \left[\mu_t(\nu_{i,t-1} + \nu_{i,t}) + \varepsilon_{i,t} - \frac{\mu_t}{\mu_{t-2}}\varepsilon_{i,t-2}\right]. \tag{12}$$

where the final term in brackets is uncorrelated with sufficiently lagged residuals. So, if $\mathrm{Cov}(\varepsilon_t, \varepsilon_{t'}) = 0$ holds for $|t' - t| \geq k$, we can obtain consistent estimates of $(\mu_t - \mu_{t-2})/\mu_{t-2}$ by estimating equation (12) via 2SLS using lags $w_{i,t-k-2}, w_{i,t-k-3}, \ldots$ as instrumental variables.

Table 1 reports estimates of skill return growth rates using equation (12) for years t covering 1979–1995, assuming that skill return growth rates are constant within two- or three-year periods (i.e., 1979–1980, 1981–1983,....1993–1995). Assuming $k = 6$, we use $(w_{i,t-8}, w_{i,t-9})$ as instruments. Table 2 reports estimates for the later years of the PSID (t covering 1996–2012) when observations become biennial.[23] In all specifications, the instruments are "strong" with very large first-stage F-statistics (for the instruments).

Panel A of Tables 1 and 2 reports estimates for the full sample of men in the PSID, while panel B reports estimates for the sample of men with 21–40 years of experience (in year t) when heterogeneity in systematic unobserved skill growth should be negligible. As might be expected from Figures 4 and 5, the estimates are quite similar in both panels, and all are negative. Many estimates in the late 1980s and 1990s are statistically significant. Panels C and D report separate estimates for non-college- and college-educated men (of all experience levels). Nearly all of these estimates are negative as well, with several statistically significant. Figure 8 combines these estimates to trace out the implied paths for μ_t from 1979 to 2012. Altogether, these results suggest that the returns to unobserved skill have declined

[23]Estimates in Table 2 assume two-year return growth rates are constant within each of the periods 1996–2000, 2002–2006, and 2008–2012, and use $(w_{i,t-8}, w_{i,t-9})$ as instruments for 1996–2000 and $(w_{i,t-8}, w_{i,t-10})$ thereafter.



(a) Non-College (b) College

Figure 7: Autocovariances for Log Wage Residuals by Educational Attainment

by roughly 50% since the mid-1980s, contrasting sharply with the sizable increase in residual inequality reported in Figure 1.

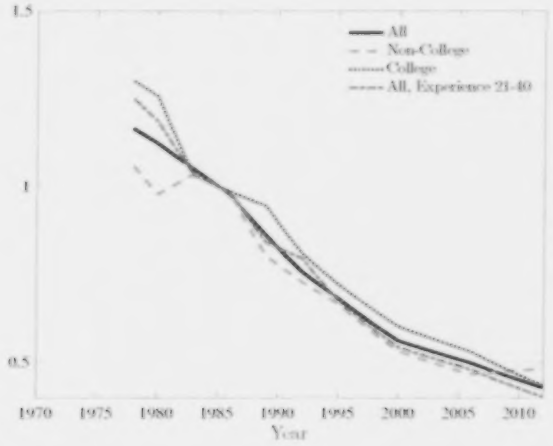


Figure 8: μ_t Implied by IV Estimates ($\mu_{1985} = 1$)

In Appendix B, we show that analogous Generalized Method of Moments (GMM) estimates to the 2SLS estimates in panel A of Tables 1 and 2 are very similar.[24] More importantly, we calculate Hansen J-statistics to test the validity of our lagged instruments, since we are overidentified when using two

[24]The GMM estimates exploit the same moments but use the optimal weighting matrix (allowing for heteroskedasticity and serial correlation within individuals).

Table 1: 2SLS estimates of $(\mu_t - \mu_{t-2})/\mu_{t-2}$ for three-year periods, 1978–1995

	1979–1980	1981–1983	1984–1986	1987–1989	1990–1992	1993–1995
			A. All men			
$(\mu_t - \mu_{t-2})/\mu_{t-2}$	-0.036	-0.044	-0.046	-0.081*	-0.082*	-0.067
	(0.045)	(0.038)	(0.038)	(0.034)	(0.035)	(0.035)
Observations	1,349	2,077	2,188	2,245	2,189	2,095
1st-Stage F-Statistic	163.09	191.61	114.85	209.42	227.13	286.96
	B. All men with 21–40 years of experience (at year t)					
$(\mu_t - \mu_{t-2})/\mu_{t-2}$	-0.052	-0.088*	-0.031	-0.100*	-0.036	-0.104*
	(0.050)	(0.043)	(0.050)	(0.046)	(0.044)	(0.045)
Observations	928	1,323	1,244	1,211	1,244	1,300
1st-Stage F-Statistic	117.23	132.19	66.26	130.53	132.83	201.62
	C. Non-college-educated men (all experience levels)					
$(\mu_t - \mu_{t-2})/\mu_{t-2}$	-0.075	0.039	-0.035	-0.127*	-0.062	-0.057
	(0.061)	(0.056)	(0.060)	(0.050)	(0.058)	(0.054)
Observations	740	1,080	997	965	897	851
1st-Stage F-Statistic	81.85	85.23	39.48	98.34	92.27	91.33
	D. College-educated men (all experience levels)					
$(\mu_t - \mu_{t-2})/\mu_{t-2}$	-0.034	-0.123*	-0.030	-0.028	-0.097*	-0.074
	(0.061)	(0.048)	(0.049)	(0.047)	(0.047)	(0.046)
Observations	508	884	1,046	1,109	1,107	1,242
1st-Stage F-Statistic	100.95	115.03	123.38	97.29	122.42	208.04

Notes: Estimates from 2SLS regression of $w_{i,t} - w_{i,t-2}$ on $w_{i,t-2}$ using instruments $(w_{i,t-8}, w_{i,t-9})$.
* Denotes significance at 0.05 level.

instruments to identify a single parameter. In all years, these J-statistics are less than 1, and we cannot reject our instruments at conventional levels. We further show that future residuals are invalid instruments (during most of our time periods), highlighting the importance of accounting for variation in skill growth. Finally, comparing estimates using only past vs. only future residuals as instruments, we show that the variance of two-year skill growth relative to prior skill levels, $\frac{\text{Var}(\nu_{t-1}+\nu_t)}{\text{Var}(\theta_{t-2})}$, ranges from 0.16 to 0.29 over our sample period.

It is worth emphasizing that these estimates require no assumptions about the variance of individual skill innovations $\nu_{i,t}$ (or non-skill shocks, $\varepsilon_{i,t}$) over time or across experience groups. The only assumptions are: (i) skill shocks $\nu_{i,t}$ are uncorrelated with past skills; and (ii) non-skill shocks $\varepsilon_{i,t}$ are uncorrelated with non-skill shocks more than five years removed, initial skill levels, and all skill shocks. (Our

Table 2: 2SLS estimates of $(\mu_t - \mu_{t-2})/\mu_{t-2}$ for four-year periods, 1996–2012

	1996–2000	2002–2006	2008–2012
	A. All men		
$(\mu_t - \mu_{t-2})/\mu_{t-2}$	-0.075*	-0.039	-0.050
	(0.025)	(0.028)	(0.027)
Observations	2,122	2,129	1,968
1st-Stage F-Statistic	369.09	344.25	341.36
	B. All men with 21–40 years of experience (at year t)		
$(\mu_t - \mu_{t-2})/\mu_{t-2}$	-0.084*	-0.040	-0.058
	(0.030)	(0.032)	(0.031)
Observations	1,427	1,591	1,493
1st-Stage F-Statistic	295.75	281.91	267.83
	C. Non-college-educated men (all experience levels)		
$(\mu_t - \mu_{t-2})/\mu_{t-2}$	-0.087*	-0.043	0.011
	(0.043)	(0.047)	(0.075)
Observations	862	826	615
1st-Stage F-Statistic	121.44	142.56	104.92
	D. College-educated men (all experience levels)		
$(\mu_t - \mu_{t-2})/\mu_{t-2}$	-0.070*	-0.041	-0.065*
	(0.031)	(0.034)	(0.029)
Observations	1,252	1,293	1,141
1st-Stage F-Statistic	260.47	218.64	229.40

Notes: Estimates from 2SLS regression of $w_{i,t} - w_{i,t-2}$ on $w_{i,t-2}$ using instruments (w_{t-8}, w_{t-9}) for 1996–2000 and (w_{t-8}, w_{t-10}) for 2002–2006 and 2008–2012. * Denotes significance at 0.05 level.

overidentification tests suggest that these assumptions cannot be rejected.) Equation (7) shows that with modest assumptions on the variance of skills and skill innovations across cohorts, additional moment restrictions can also be used to help identify μ_t in all but the final few years.

We have, thus far, used a very limited set of lagged residuals as instruments to keep the specifications similar across years and to allow estimation of skill return growth rates back to 1979. One could certainly add more lags as instruments for most years. Rather than report several sets of 2SLS estimates, the next section employs a minimum distance estimator to estimate the returns to unobserved skills, as well as the variances of unobserved skills and shocks over time, using the full set of available moments in the data. "Smoothness" restrictions on the time and experience patterns for skill and non-skill shock variances are also imposed. The combination of more moments and modest "smoothness" restrictions considerably improves precision, yet it produces very similar estimates for the time sequence of μ_t.

5 Minimum Distance Estimation

We now estimate the full model by choosing parameters to minimize the distance between the sample covariances and the theoretical covariances implied by the model. This requires a more complete specification of the model; however, since we are focused on estimating the evolution of unobserved skill returns and the variances of skills over time, we need not model or use higher order moments.

5.1 Specification and Identification

We assume that individuals enter the labor market with initial skill ψ_i and no other prior shocks, which implies that unobserved skill in year t for individual i who started working in year c_i can be written as

$$\theta_{i,t} = \psi_i + \sum_{j=0}^{t-c_i-1} \nu_{i,t-j}. \tag{13}$$

We begin by assuming that skill growth shocks, $\nu_{i,t}$, are serially uncorrelated but relax this assumption later in Subsection 5.3. As in much of the literature, we assume that the transitory component $\varepsilon_{i,t}$ is a moving average process with order q:

$$\varepsilon_{i,t} = \sum_{j=0}^{\min\{q,t-c_i-1\}} \beta_j \xi_{i,t-j}, \tag{14}$$

where $\beta_0 = 1$.

Given this MA(q) specification for ε_{it}, identifying assumption (i) in Proposition 1 (i.e., $\text{Cov}(\varepsilon_t, \varepsilon_{t'}|c) = 0$ for $|t' - t| \geq k$) holds for $k = q + 1$. This demonstrates identification for all skill returns μ_t (up to one normalization) as well as cohort- and time-specific variances of skills, $\text{Var}(\theta_t|c) = \text{Cov}(w_t, w_{t'}|c)/\mu_t\mu_{t'}$, for cohorts observed in years t and any $t' \geq t+q+1$. Next, $\text{Cov}(\varepsilon_t, \varepsilon_{t'}|c) = \text{Cov}(w_t, w_{t'}|c) - \mu_t\mu_{t'}\text{Var}(\theta_t|c)$ for $t' \geq t$ is identified for (c, t) such that $\text{Var}(\theta_t|c)$ is identified. Then, β_j is identified from some cohort c and period $t = c + 1$ as follows:

$$\frac{\text{Cov}(\varepsilon_t, \varepsilon_{t+j}|c)}{\text{Var}(\varepsilon_t|c)} = \beta_j \frac{\text{Var}(\xi_t|c)}{\text{Var}(\xi_t|c)} = \beta_j.$$

Given β_j's, we can recover $\text{Var}(\xi_t|c)$ each period by following cohorts over time.[25]

In our empirical analysis, we assume that the initial skill variance for each cohort $\text{Var}(\psi|c)$ is a cubic polynomial in c. We also assume that the variances of shocks for each period and cohort can be written as products of time trends and experience trends:

$$\text{Var}(v_t|c) = \pi(t)\phi(e) \qquad \text{and} \qquad \text{Var}(\xi_t|c) = \omega(t)\kappa(e),$$

where $e = t - c$ is potential work experience and we normalize $\phi(20) = \kappa(20) = 1$. We assume that the experience trends $\phi(e)$ and $\kappa(e)$ are quadratic. The time trend for the skill shock variance $\pi(t)$ is assumed to be cubic, but the time pattern for the variances of transitory shocks $\omega(t)$ is unrestricted. Finally, we assume that the variances of all shocks prior to 1970 are the same as in 1970.

We estimate μ_t for all t (normalizing $\mu_{1985} = 1$), $\{\beta_j\}_{j=1}^q$, cohort trends in $\text{Var}(\psi|c)$, time trends in shocks, $\pi(t)$ and $\omega(t)$, and experience trends in shocks, $\phi(e)$ and $\kappa(e)$. Since the relative wages between those who did not attend college and those who did have diverged significantly during the sample period (Katz and Murphy, 1992), we also estimate all parameters separately for the two groups, which we call "sectors". Let s_{it} be an indicator variable for college attendance.

For a given parameter vector $\mathbf{A}$, we can compute theoretical counterparts for $\text{Cov}(w_t, w_{t'}|s, c)$ implied by the model (2), (13), and (14) and compare them with the sample covariances. Since some cohort (or, equivalently, experience $e = t - c$) cells have few observations when calculating residual covariances, we partition the experience set $E = \{1, \ldots, 40\}$ into 10-year experience groups E_1, E_2, E_3, and E_4, corresponding to 1–10, 11–20, 21–30, and 31–40 years, respectively, and aggregate within experience groups.

[25] Consider a cohort c. For the initial period $t = c + 1$, $\text{Var}(\xi_t|c) = \text{Var}(\varepsilon_t|c)$. For $t > c + 1$,

$$\text{Var}(\xi_t|c) = \text{Var}(\varepsilon_t|c) - \sum_{j=1}^{\min\{q, t-c-1\}} \beta_j^2 \text{Var}(\xi_{t-j}|c).$$

The minimum distance estimator $\hat{\mathbf{A}}$ solves

$$\min_{\mathbf{A}} \sum_{(s,j,t,t')\in\Gamma} \left\{\widehat{\text{Cov}}(w_t, w_{t'}|s, E_j) - \text{Cov}(w_t, w_{t'}|s, E_j, \mathbf{A})\right\}^2,$$

where $\Gamma = \{s, j, t, t' | 1970 \leq t' \leq t \leq 2012, s \in \{0, 1\}, j \in \{1, 2, 3, 4\}\}$, $\widehat{\text{Cov}}(w_t, w_{t'}|s, E_j)$ is the sample covariance conditional on sector s and experience group E_j in year t, and $\text{Cov}(w_t, w_{t'}|s, E_j, \mathbf{A})$ is the corresponding theoretical covariance given parameter $\mathbf{A}$.

5.2 Estimation Results

We discuss results for log wage residuals with MA(5) transitory shocks in the text, consistent with $k = 6$ assumed in Section 4; however, conclusions are quite similar for log annual earnings and other specifications for the transitory component.[26]

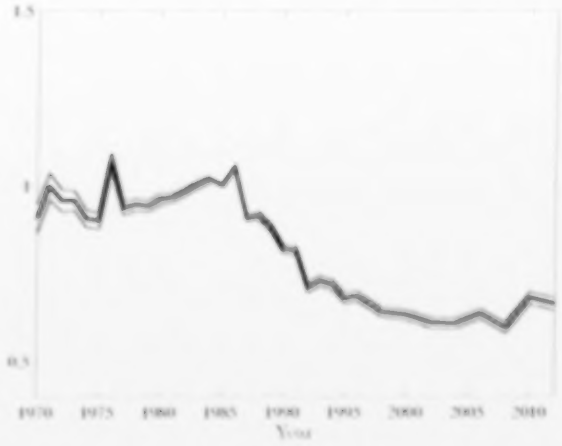
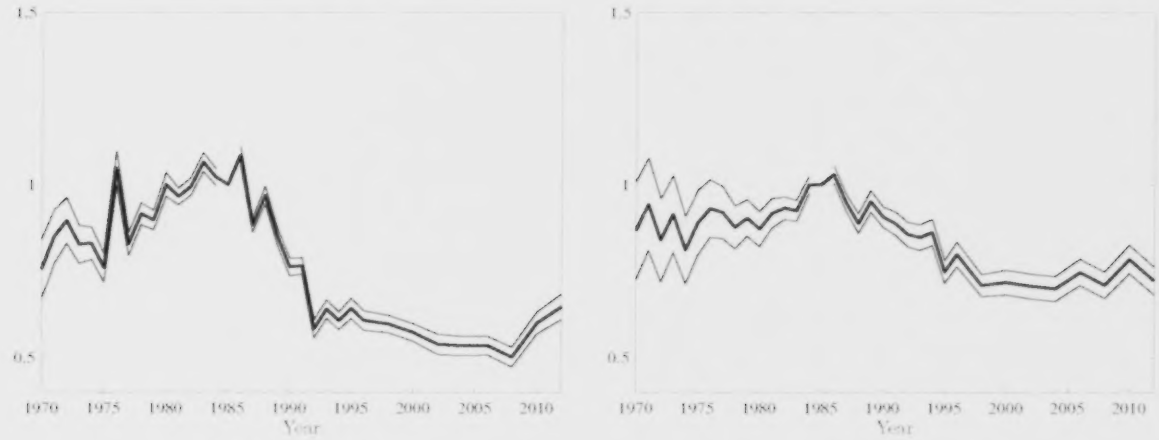


Figure 9: Estimated μ_t (Thick Lines) with 95% Confidence Interval (Thin Lines)

Estimated Returns Figure 9 reports the estimated returns over time along with their 95% confidence intervals. The returns are quite similar to those reported in Figure 8 but are much more precisely estimated now. Figure 10 shows separate estimates by education sector. Overall, the returns to unobserved skills fell substantially after 1985 in both sectors despite the increasing residual inequality during the period. Although the returns for non-college- and college-educated workers display qualitatively similar time patterns, the return for non-college-educated workers declines about 20 percentage points more between

[26] Available upon request.



(a) Non-College (b) College

Figure 10: μ_t Estimated Separately by Educational Attainment

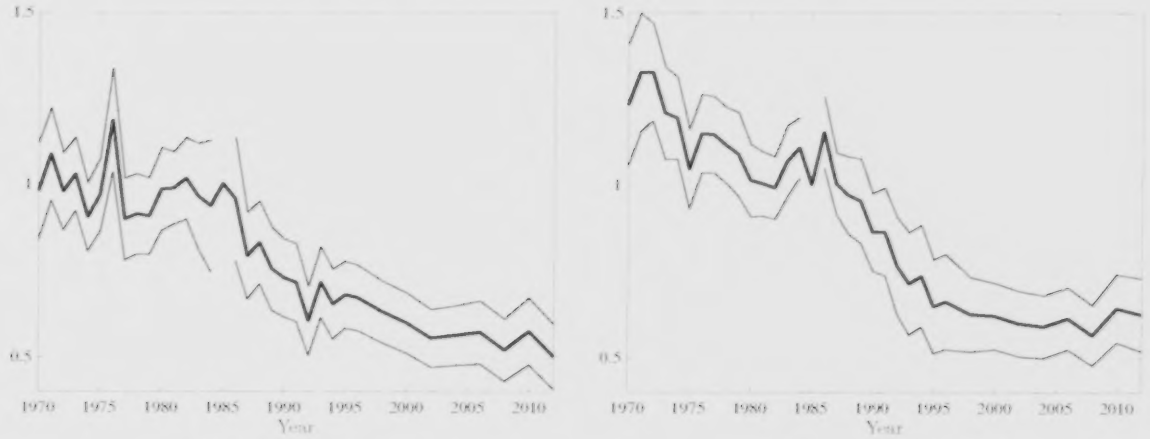
1985 and 2008 than that of college-educated workers. Indeed, we can reject the hypothesis that the returns to unobserved skills are identical across education groups at the 5% significance level.[27]

One potential concern is that skill growth shocks may be serially correlated, especially for younger workers who are likely to be investing in their skills through job training. This is less of a concern for older workers, whose wage profiles are relatively flat due to weaker incentives for investment (e.g., Heckman, Lochner, and Taber, 1998; Huggett, Ventura, and Yaron, 2011). Figure 11 shows that estimated returns to unobserved skill are quite similar for workers with low (1–20 years) and high (21–40 years) levels of experience, alleviating concerns that our estimated decline in μ_t over time is due to serially correlated skill shocks.[28] In Subsection 5.3, we further show that the estimated μ_t series is quite similar to that of Figure 9 when we explicitly model individual heterogeneity in systematic life cycle wage growth profiles.

Our finding that the returns to unobserved skills have fallen since the mid-1980s is consistent with the findings of Castex and Dechter (2014), who show that the returns to cognitive ability as measured by AFQT scores fell in the 2000s relative to the 1980s by comparing NLSY79 and NLSY97 cohorts. The decreasing returns to unobserved skills differ, however, from the conclusions reached in the literature based on the Current Population Survey (CPS) data, which typically equates changes in the total variance of log wage residuals with changes in the returns to unobserved skills. This literature generally concludes that the returns to unobserved skill increased steadily after the early 1970s (Juhn, Murphy, and Pierce,

[27] A Wald test is used to test the null hypothesis that μ_t is identical across sectors for all $t \neq 1985$.

[28] These estimated return sequences are based on the full sample assuming parameters are the same across education groups (as in Figure 9) but allowing the μ_t returns to depend on worker experience in period t.



(a) Experience 1–20 Years

(b) Experience 21–40 Years

Figure 11: μ_t Estimated Separately by Experience

1993; Katz and Autor, 1999; Acemoglu, 2002).[29]

The PSID-based literature on earnings dynamics focuses primarily on the relative importance of permanent and transitory shocks, typically ignoring variation in the returns to unobserved skills.[30] Haider (2001) and Moffitt and Gottschalk (2012) are notable exceptions. Haider (2001) estimates that the returns to unobserved skills were stable in the 1970s and then increased throughout the 1980s, while Moffitt and Gottschalk (2012) find that the returns increased until the mid-1980s, stabilized, and then increased again in the mid-1990s.[31]

Of course, the patterns estimated in the earnings dynamics literature may differ from ours due to labor supply responses and the distinction between annual earnings and hourly wages. This is not the entire story, however. Figure 12 shows that the evolution of estimated returns to skill in Haider (2001) and Moffitt and Gottschalk (2012) differs from ours largely because they restrict the variance of permanent skill shocks to remain constant over time. The estimates in this figure are based on an analysis of log hourly wage residuals that pools non-college- and college-educated men, estimating a single series for μ_t as in Figure 9 (referred to as "Baseline" in the figure). The blue line with circles reports estimates

[29]Lemieux (2006) is a notable exception. After controlling for composition effects, he estimates that the returns to unobserved skill declined slightly in the 1970s and 1990s.

[30]See, among others, Abowd and Card (1989); Blundell and Preston (1998); Meghir and Pistaferri (2004); Blundell, Pistaferri, and Preston (2008); Heathcote, Perri, and Violante (2010). See MaCurdy (2007) for a review.

[31]Other studies exploit different panel data sets on earnings to estimate similar models to Haider (2001) and Moffitt and Gottschalk (2012). DeBacker et al. (2013) use U.S. tax return data from 1987 to 2009, while Baker and Solon (2003) exploit Canadian tax return data from 1976 to 1992. Both studies reach similar conclusions that the returns to skill increased over time.

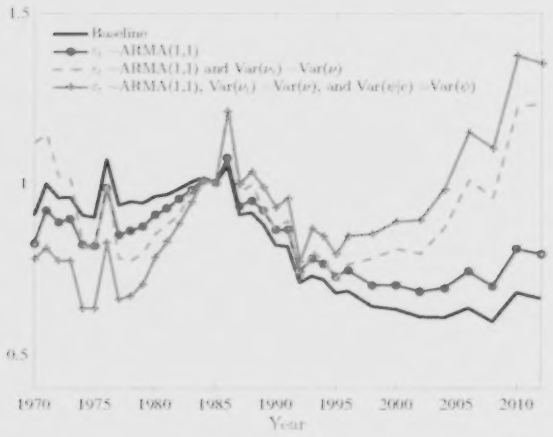


Figure 12: Estimated μ_t Under Different Restrictions

assuming that transitory non-skill shocks follow an ARMA(1,1) (rather than MA(5)) process as in their analyses. The estimated time patterns for μ_t are quite similar to our baseline estimates. More generally, different assumptions about the persistence of transitory non-skill shocks all lead to similar conclusions about the returns to skill. The red dashed line reports estimates additionally assuming that the variance of skill shocks is time invariant, while the green line with + signs further assumes that the initial skill distribution is identical across cohorts. The last model is nearly identical to those estimated in Haider (2001) and Moffitt and Gottschalk (2012), and the estimated time series for μ_t is similar to theirs.[32] Moving from the baseline to the final model, the estimated returns rotate counter-clockwise, generating strong positive trends both before the mid-1980s and after the mid-1990s. The difference between the ARMA(1, 1) model with time-varying vs. time-invariant skill growth shock distributions is dramatic after 1990, highlighting the importance of accounting for the rise in the variance of skill growth innovations. When the variance of skill growth shocks is not allowed to increase over time, the model is "forced" to explain the increasing residual variance via an increase in the returns to skill.

Variance of Unobserved Skills and the Rising Residual Variance The fact that estimated returns have evolved quite differently from residual inequality suggests that the role of unobserved skills might also have changed. Figure 13 decomposes the residual variance into two components: the unobserved skill component ($\mu_t\theta_t$) and transitory component (ε_t).[33] Initially quite low, the variance of the unobserved

[32]These studies also allow for heterogeneity in the growth rate of unobserved skills. We incorporate this heterogeneity in Subsection 5.3 and show that the estimated returns to skill are quite similar to our "Baseline" series.

[33]As shown in Section 2, the variances of unobserved skills and transitory components are not nonparametrically identified for the last few years of our panel. In this figure and those that follow, we report only distributions of skills and shocks through 2006.

skill component rises over the 1970s and early 1980s before stabilizing after 1985. The variance of the transitory component rises in the late 1980s and early 1990s. The unobserved skill component explains about 65% of the total residual variance at its peak in the late 1980s with its share decreasing thereafter.

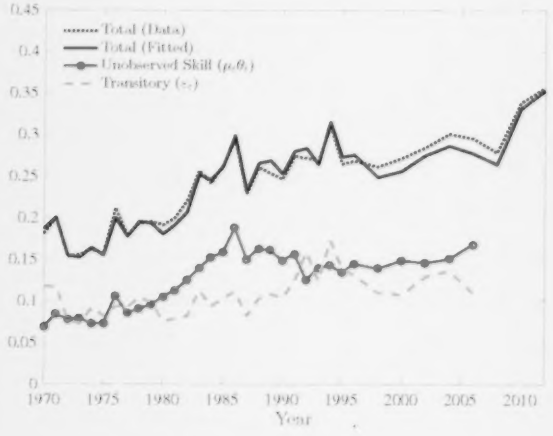


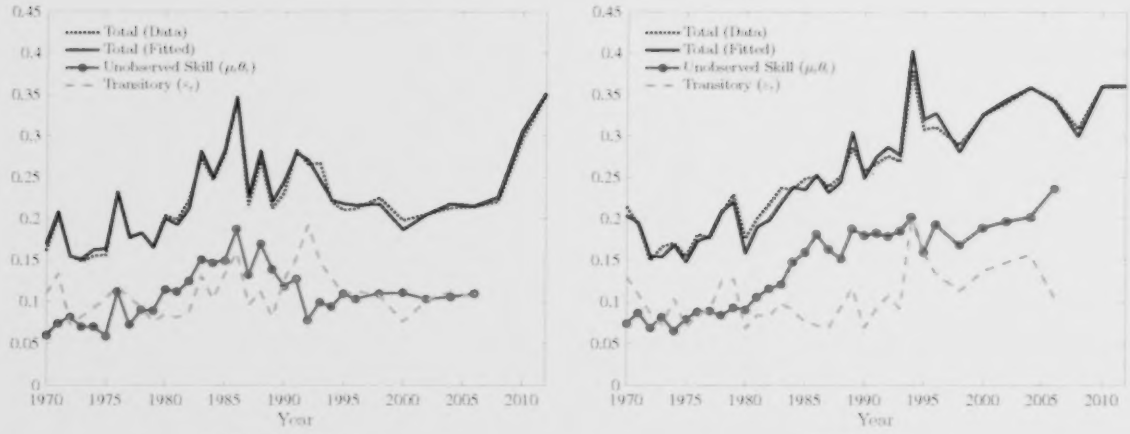
Figure 13: Log Wage Residual Variance Decomposition (Full Sample)

Figure 14 decomposes the residual variance when estimating the model separately for non-college- and college-educated men. The figure reveals that the general time patterns for overall residual inequality (within education groups) are driven largely by changes in the variance of the unobserved skill component, $\mu_t\theta_t$. The variance of this component rose for both college- and non-college-educated workers over the 1970s and early 1980s; however, it reversed course for non-college-educated workers over the late 1980s and early 1990s. Among college-educated workers, the variance of the unobserved skill component continued to rise throughout the sample period, although there was a slowdown in the growth rate beginning in the mid-1980s. The course reversal for the $\text{Var}(\mu_t\theta_t)$ among non-college-educated workers and the slowdown for college-educated workers beginning in the mid-1980s is driven entirely by the sudden and lasting decline in skill returns μ_t documented in Figure 10. As Figure 15 shows, variation in skill levels θ_t rose continually from the mid-1980s through the early 2000s.

Figure 15 also decomposes the rising variance in unobserved skills θ_t into the variance of initial skill levels and the accumulation of all permanent skill shocks.[34] The increasing variance of unobserved skills is driven entirely by an increase in the variability of skill growth shocks over time. The estimated time trends for the variance of skill growth shocks, $\pi(t)$, is reported in Figure 16.[35]

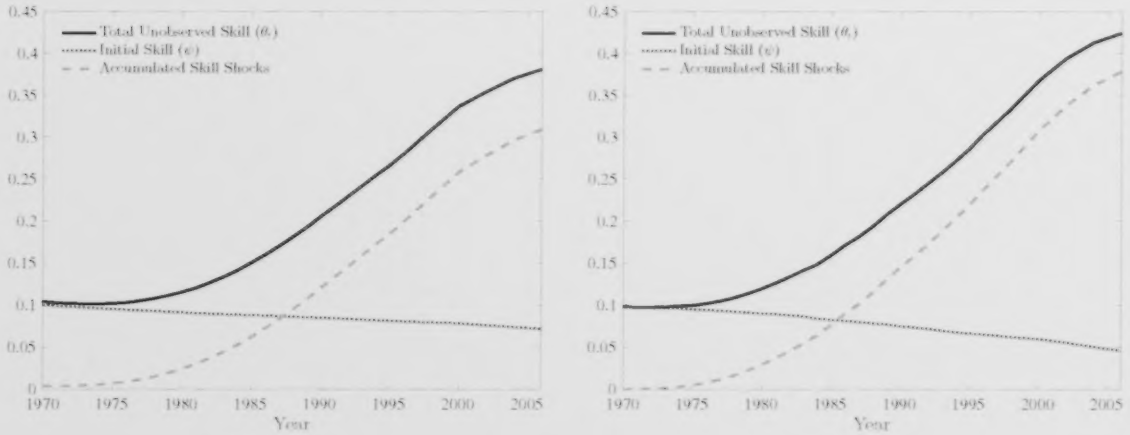
[34]These aggregate across all cohort/experience groups within each period. Initial skills are given by ψ for each cohort, while the accumulation of permanent skills are simply $\theta_t - \psi$ for each cohort.

[35]See Appendix E for estimated cohort trends in the variance of initial skills (Figure E-3) and for the experience profiles for the variance of permanent skill shocks, $\phi(e)$ (Figure E-4).



(a) Non-College (b) College

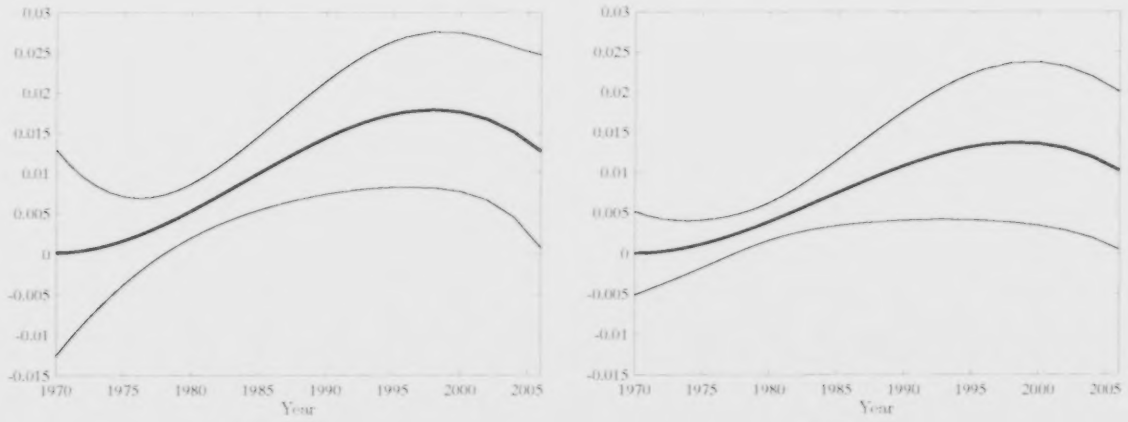
Figure 14: Log Wage Residual Variance Decomposition Separately by Education



(a) Non-College (b) College

Figure 15: Skill Variance Decomposition by Education

Summarizing, we find that U.S. trends in residual wage inequality closely resemble trends in the variance of the unobserved skill component of wages. Variation in the total value of unobserved skill, $\mu_t\theta_t$, has risen considerably since 1970. For both college- and non-college-educated men, the increase was strongest before the mid-1980s, after which it increased more slowly for college-educated men and declined for almost a decade among non-college-educated men. The increasing variability in the total value of skill contrasts sharply with the strong decline in returns to unobserved skill, μ_t, from the



(a) Non-College (b) College

Figure 16: Time Trends in the Variances of Skill Growth Shocks, $\pi(t)$, by Education

mid-1980s to late 1990s, especially among non-college-educated men. This decline in returns is largely responsible for the slowdown (college-educated men) and roughly decade-long fall (non-college-educated men) in residual inequality; yet, it was offset by a strong increase in the variance of unobserved skill θ_t beginning in the early 1980s. Finally, the increasing variance of skills is driven by increasing variation in skill growth shocks, ν_t, over time rather than an increase in initial skill levels for more recent cohorts. The transitory, non-skill component of wages, ε_t, showed little systematic growth over this period.[36]

5.3 Accounting for Heterogeneity in Systematic Life Cycle Skill Growth

We now incorporate potential heterogeneity in systematic life cycle skill growth as discussed in Section 2.2.2. In particular, we consider a flexible process governing heterogeneity over the life cycle and across time: $\nu_{i,t} = \tau_t(c_i)\delta_i + \tilde{\nu}_{i,t}$.

For practical purposes, we make a few additional assumptions on the structure of unobserved skill growth in estimating this framework using the PSID. We begin by assuming that $\tau_t(c)$ is separable in experience and time, so $\tau_t(c) = \chi(t)\eta(e)$ where $e = t - c$. The parameter $\chi(t)$ captures any time-varying differences in systematic skill growth (normalizing $\chi(1985) = 1$), while $\eta(e)$ allows skill growth heterogeneity to vary with experience. We assume that $\chi(t)$ is a cubic polynomial in time, while we assume that $\eta(e)$ declines linearly in experience (a natural assumption, given the decline in skill growth over the life cycle). Specifically, we assume $\eta(e) = \max\{1 - e/\bar{e}, 0\}$ with $\bar{e} = 30$. Finally, we assume that $\text{Var}(\delta|c)$ and $\text{Cov}(\delta, \psi|c)$ are the same across cohorts, while we continue to allow for cohort trends

[36] See Figures E-5 and E-6 in Appendix E for estimated time and experience patterns for transitory shocks.

in $\text{Var}(\psi|c)$ as above.

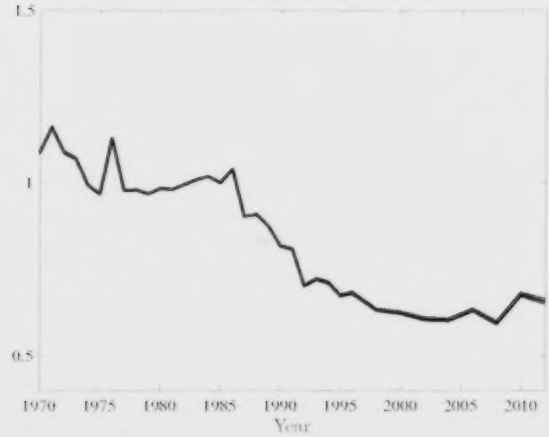
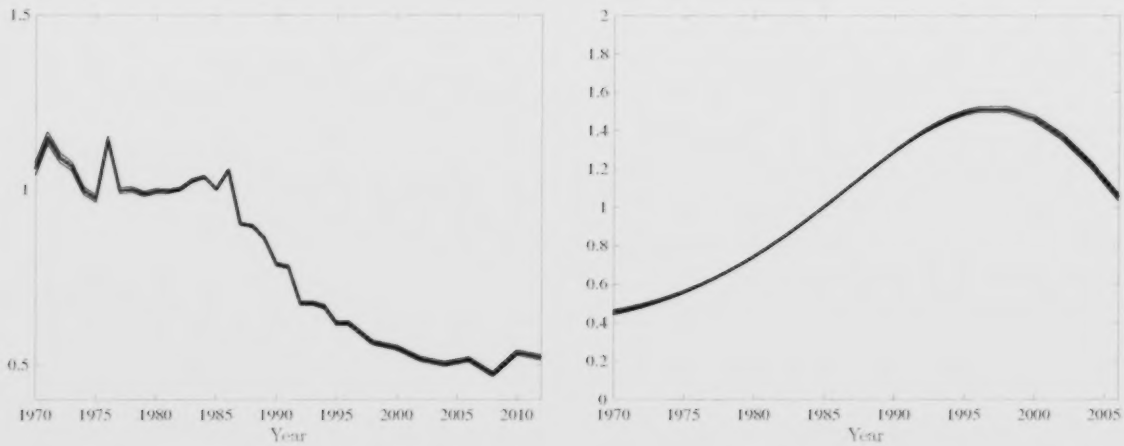


Figure 17: Estimated μ_t Accounting for Heterogeneous Skill Growth with $\chi(t) = \chi$



(a) μ_t (b) $\chi(t)$

Figure 18: Estimated μ_t and $\chi(t)$ Accounting for Time-Varying Variance of Heterogeneous Skill Growth

We first examine the impacts of introducing systematic skill growth heterogeneity for our estimates of skill returns, keeping $\chi(t) = \chi$ fixed over time. This is similar to previous studies that allow for HIP; however, we continue to allow for time variation in the variance of skill shocks $\tilde{v}_t$.[37] Figure 17 shows that incorporating systematic skill growth heterogeneity has very little effect on our estimated μ_t skill return

[37]For this analysis, we pool all men and assume parameters are the same across education groups (college and non-college).

series (compare with Figure 9). Allowing $\chi(t)$ to vary over time (Figure 18), we estimate substantial growth in the variance of systematic skill growth over the 1980s and 1990s.These results suggest a slightly stronger decline in skill returns after 1985.

Finally, Figure 19 shows the dramatic increase in the variance of unobserved skills over time and decomposes this variance into the part due to heterogeneity in initial skills, ψ, which varies across cohorts; the part due to systematic skill growth (including terms related to $\text{Var}(\delta)$ and $\text{Cov}(\psi, \delta)$ in equation (9)); and the part due to accumulated idiosyncratic skill growth shocks, $\tilde{v}_t$. Consistent with our earlier results, variation in initial skills plays a minor role in the rising skill variance. Instead, we observe strong increases in the variances of both systematic and idiosyncratic skill growth innovations.

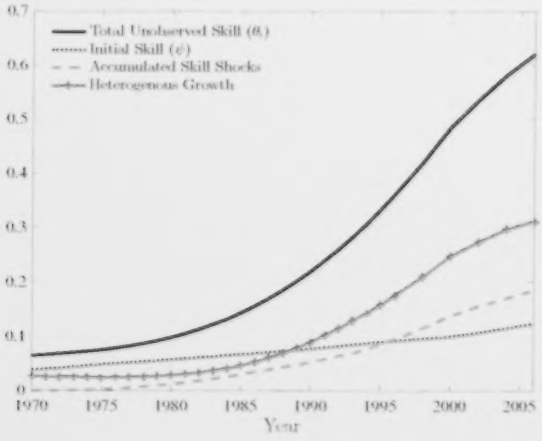


Figure 19: Skill Variance Decomposition Accounting for Time-Varying Variance of Heterogeneous Skill Growth

6 Interpreting Skill Returns in a Demand and Supply Framework

In this section, we examine the extent to which the estimated widening of the skill distribution, as well as changes in the demand for skill (due to changes in technology), has contributed to the falling return to skill since the mid-1980s. In particular, we assess the contributions of skill demand and supply using an equilibrium framework based on the assignment model of Sattinger (1979). Assignment models are particularly useful for studying within-group wage inequality because they generate a hedonic wage function that is non-linear in skill.[38] The framework we develop produces log wage specifications identical

[38]See Sattinger (1993) for an early review. Recent theoretical and empirical studies of income inequality based on this framework include Tervió (2008); Gabaix and Landier (2008); Costinot and Vogel (2010); Lindenlaub (forthcoming); Burstein, Morales, and Vogel (2015); Ales, Kurnaz, and Sleet (2015); Scheuer and Werning (2015).

to those estimated earlier in the paper and provides results that are useful for recovering changes in skill demand from our estimated returns and distributions of unobserved skills.

In the model, the returns to skill are generated by differences in the productivity of skill and amplified by differences in the productivity of jobs or tasks in which skills are employed. Heterogeneity in job/task productivity may be purely technological with more complex tasks performed more efficiently by more skilled workers. Heterogeneous productivity across jobs might also derive from differences in the amount of resources under each worker's control, such as the amount of capital or the number of reporting workers in a hierarchy. Differences could also arise from different vintages of technology embodied in capital. As in the models of "span-of-control" (Lucas, 1978; Rosen, 1982) and "superstars" (Rosen, 1981), the production technology determines the extent to which differences in skills are magnified to differences in earnings. When jobs are in fixed supply, the distribution of skills also affects the return to skills through its impact on the equilibrium assignment of skills and jobs.

Since the aim of this analysis is to understand the evolution of skill returns in equilibrium, we do not consider the economic forces driving changes in the underlying distribution of skills. We also abstract from transitory wage shocks that are unrelated to skills.

6.1 Assignment Model of Labor Market

Endowment We consider an economy populated by a continuum of measure 1 of workers and jobs. In each period t, there exist workers endowed with heterogeneous skills Θ_t and jobs with heterogeneous productivity Z_t. Worker skills and job productivities are continuously distributed with full support on the real line according to distribution functions $F_t(\Theta_t)$ and $G_t(Z_t)$. Skill Θ_t is assumed to be observed by all market participants.[39]

Technology Production takes place through one-to-one matching between workers and jobs. If a worker with skill Θ_t works at a job with productivity Z_t, $Y_t(\Theta_t, Z_t) \geq 0$ units of output are produced. We assume that $Y_t(\cdot, \cdot)$ is twice continuously differentiable, strictly increasing, and satisfies the following strict supermodularity condition:

$$\frac{\partial^2 Y_t(\Theta_t, Z_t)}{\partial \Theta_t \partial Z_t} > 0. \tag{15}$$

These assumptions imply that high-skill workers are more productive (i.e., they have absolute advantages) than low-skill workers at all jobs, but the productivity gap between high- and low-skill workers

[39]It need not be perfectly observed by the econometrician. As discussed in Subsection 6.3, the component of skill that is orthogonal to observed variables, $\theta_t \equiv \Theta_t - \mathrm{E}[\Theta_t | \boldsymbol{x}_t]$, corresponds to unobserved skill in the empirical model studied earlier.

is greater at more productive jobs due to complementarity between workers and jobs. Therefore, the efficient assignment that maximizes aggregate output features positive assortative matching where more skilled workers work at more productive jobs (Becker, 1973).

Profit Maximization All markets are perfectly competitive. Workers with skill Θ_t earn $W_t(\Theta_t)$ and output is sold at a price normalized to 1. Producers maximize profits, solving

$$\Pi_t(Z_t) \equiv \max_{\Theta_t} \left\{ Y_t(\Theta_t, Z_t) - W_t(\Theta_t) \right\}. \tag{16}$$

Denote the solution by $\hat{\Theta}_t(Z_t)$. Because it is strictly increasing in Z_t due to the strict supermodularity of $Y_t(\cdot,\cdot)$, its inverse $\hat{Z}_t(\Theta_t)$, which we call the "matching function," is well defined. The necessary first-order condition for profit maximization is

$$\frac{\partial Y_t(\Theta_t, \hat{Z}_t(\Theta_t))}{\partial \Theta_t} = \frac{dW_t(\Theta_t)}{d\Theta_t}. \tag{17}$$

The second-order condition is satisfied at optimum due to the supermodularity.

Market Clearing The labor market clears if, for all Θ_t, the fraction of producers demanding skills Θ_t or less equals the fraction of workers supplying skills Θ_t or less:

$$F_t(\Theta_t) = G_t(\hat{Z}_t(\Theta_t)). \tag{18}$$

With one-to-one matching, the equilibrium assignment is entirely characterized by the market clearing condition, independent of technology.

Given the equilibrium assignment defined by the market clearing condition (18), the condition for profit maximization (17) determines the slope of the hedonic wage function. To pin down the level of wages, we assume that both producers and workers have an option not to engage in production, in which case they earn zero, and that the least productive worker-job pair produces nothing (i.e., $\lim_{\Theta_t \to -\infty} Y_t(\Theta_t, \hat{Z}_t(\Theta_t)) = 0$). Then, it can be shown that, in equilibrium, all workers participate, and the least skilled worker earns zero: $\lim_{\Theta_t \to -\infty} W_t(\Theta_t) = 0$.

Equilibrium Wage A competitive equilibrium consists of the matching function $\hat{Z}_t(\Theta_t)$ and the wage function $W_t(\Theta_t)$ that satisfy the first-order condition for profit maximization (17) and the market clearing condition (18). The equilibrium wage for a worker with skill Θ_t, $W_t(\Theta_t)$, is given by the solution to differential equation (17) with the condition $\lim_{\Theta_t \to -\infty} W_t(\Theta_t) = 0$.

We can rearrange the first-order condition (17) to derive the following expression for the equilibrium return to skill:[40]

$$\underbrace{\frac{d \ln W_t(\Theta_t)}{d\Theta_t}}_{\text{Return to skill}} = \underbrace{\frac{\partial \ln Y_t(\Theta_t, \hat{Z}_t(\Theta_t))}{\partial \Theta_t}}_{\text{Partial elasticity of output}} \underbrace{\left(\frac{W_t(\Theta_t)}{Y_t(\Theta_t, \hat{Z}_t(\Theta_t))}\right)^{-1}}_{\text{Labor share}}. \tag{19}$$

The return to skill consists of two components. The first term, which we call the "partial elasticity of output," is the proportional change in output associated with a marginal skill change, holding productivity of a job constant. The second term is the inverse of the worker's share of revenue, or labor share. Therefore, the return to skill for a worker is determined by the worker's marginal contribution to output (relative to a marginally less skilled worker) as well as the share of output that accrues to the worker.

6.2 Cobb-Douglas Technology and Returns to Skill

When the production function is Cobb-Douglas and skills and job productivities are normally distributed, the return to skill (19) is identical across skill levels, consistent with our empirical model.[41] Proposition 2 shows that the return to skill can be derived in closed form as a function of production technology parameters and the variances of both skills and job productivity. (Its proof is provided in Appendix C.)

Proposition 2. *Suppose that (i)* $\ln Y_t(\Theta_t, Z_t) = \lambda_t \Theta_t + \gamma_t Z_t$ *and (ii)* Θ_t *and* Z_t *are normally distributed. Then, the equilibrium return to skill is given by*

$$\frac{d \ln W_t(\Theta_t)}{d\Theta_t} = \lambda_t + \gamma_t \frac{\sigma(Z_t)}{\sigma(\Theta_t)}, \tag{20}$$

where $\sigma(X)$ *is the standard deviation of* X.

It is useful to provide some intuition for this result. Because the technology ensures that wages are a constant fraction of output, both wages and output will increase by the same proportion with an increase in skills.[42] For simplicity, we consider the effects of skills on output. More skilled workers produce more at all jobs, which is captured by the partial elasticity of output λ_t. They also work at jobs with higher productivity. Taking into account this sorting effect, the proportional increase in output for higher-skilled workers, which we call the "total elasticity of output," is

$$\frac{d \ln Y_t(\Theta_t, \hat{Z}_t(\Theta_t))}{d\Theta_t} = \frac{\partial \ln Y_t(\Theta_t, \hat{Z}_t(\Theta_t))}{\partial \Theta_t} + \frac{\partial \ln Y_t(\Theta_t, \hat{Z}_t(\Theta_t))}{\partial Z_t} \frac{d\hat{Z}_t(\Theta_t)}{d\Theta_t} = \lambda_t + \gamma_t \frac{d\hat{Z}_t(\Theta_t)}{d\Theta_t}. \tag{21}$$

[40] Costinot and Vogel (2010) and Sampson (2014) also derive the same expression in one-to-many matching models.

[41] Normality is not central to the results of this subsection, although it is a natural assumption given the normality of log wages. All of the results in this subsection hold more generally when Θ_t and Z_t belong to the same location-scale family distribution, which also includes uniform, logistic, Cauchy, extreme value, and 2-parameter exponential distributions.

[42] A constant labor share results from the constant marginal rate of technical substitution between Θ_t and Z_t.

The normality assumption implies that $d\hat{Z}_t(\Theta_t)/d\Theta_t = \sigma(Z_t)/\sigma(\Theta_t)$, so this expression is identical to that of equation (20).

Combining equations (19) and (20), we get the following formula for the labor share:

$$\alpha_t \equiv \frac{W_t(\Theta_t)}{Y(\Theta_t, \hat{Z}_t(\Theta_t))} = \frac{\lambda_t}{\lambda_t + \gamma_t \frac{\sigma(Z_t)}{\sigma(\Theta_t)}}. \tag{22}$$

Intuitively, the labor share equals the ratio between the partial and total elasticity of output to skill and reflects the worker's direct marginal contribution to output relative to the total increase in output (including any improvement in job productivity).

Equation (20) reveals that technology, λ_t and γ_t, as well as the relative heterogeneity between workers and jobs, is important for the equilibrium return to skill.[43] When workers are relatively homogeneous compared with jobs, the matching function is steep, and a slightly more skilled worker is assigned to a much more productive job. This generates large proportional changes in output and wages, amplifying the innate skill differences across workers. By contrast, when jobs are relatively homogeneous compared to workers, the sorting effect is small, and the differences in wages reflect mainly skill differences.

The Cobb-Douglas case provides a useful framework for studying the key forces that determine the return to skill. It also provides a direct mapping to our empirical model in which log wages are linear in skills.

6.3 Demand and Supply Factors and their Contributions to Skill Returns

We now use the framework from the previous subsection, along with our minimum distance estimation results from Section 5, to identify changes in demand and supply factors over time, as well as their contributions to the changes in estimated skill returns.

Equation (20) implies that our estimated skill returns are determined by $\mu_t = \lambda_t + \gamma_t \frac{\sigma(Z_t)}{\sigma(\Theta_t)}$. Our estimated variances of unobserved skill can be used to determine the evolution of the supply factor $\sigma(\Theta_t)$ over time. Using additional data on labor compensation and firm output over time to calculate labor shares, α_t, we can also identify two distinct demand forces. First, we can use equation (22) to recover the partial elasticity of output with respect to skill over time, $\lambda_t = \alpha_t \mu_t$, from the labor shares and the estimated returns to skill. Second, we can use the estimated skill variances, along with μ_t and λ_t, to obtain $\gamma_t \sigma(Z_t) = (\mu_t - \lambda_t)\sigma(\Theta_t)$. This reflects the partial elasticity of output with respect to scaled job productivity, $Z_t/\sigma(Z_t)$. Without additional information, we cannot distinguish between changes in

[43] Sattinger (1980) and Gabaix and Landier (2008) derive skill returns similar to equation (20); however, they consider cases in which the minimum wage level is not zero, so skill returns are not necessarily independent of skills. Gabaix and Landier (2008) also assume that Θ_t and Z_t follow a 2-parameter exponential distribution rather than a normal distribution.

productivity differences across specific jobs or tasks vs. changes in the distribution of job/tasks.[44]

We assume that the labor market is segmented between a sector for college-educated workers and another sector for non-college-educated workers, allowing all parameters to differ across sectors. We therefore use the baseline results estimated separately by sector (i.e., μ_t from Figure 10 and $\text{Var}(\theta_t)$ from Figure 15).

Supply Factor We assume that skills are correlated with the workers' observed characteristics $\boldsymbol{x}_{i,t}$ but are not fully observed by an econometrician. Within each (college/non-college) sector, we assume that skill for individual i in year t is

$$\Theta_{i,t} = g_t(\boldsymbol{x}_{i,t}) + \theta_{i,t},$$

where $\text{E}[\theta_t|\boldsymbol{x}_t] = 0$. From equation (20), the equilibrium wage function can be written as $\ln W_t(\Theta_{i,t}) = \zeta_t + \mu_t\Theta_{i,t}$ for some time-varying constant ζ_t. Assuming that log wages are the sum of a skill-related component that reflects the equilibrium wage function and a transitory non-skill component, $\varepsilon_{i,t}$, they can be written as

$$\ln W_{i,t} = \zeta_t + \mu_t\Theta_{i,t} + \varepsilon_{i,t} = \underbrace{\zeta_t + \mu_t g_t(\boldsymbol{x}_{i,t})}_{=f_t(\boldsymbol{x}_{i,t})} + \underbrace{\mu_t\theta_{i,t} + \varepsilon_{i,t}}_{=w_{i,t}},$$

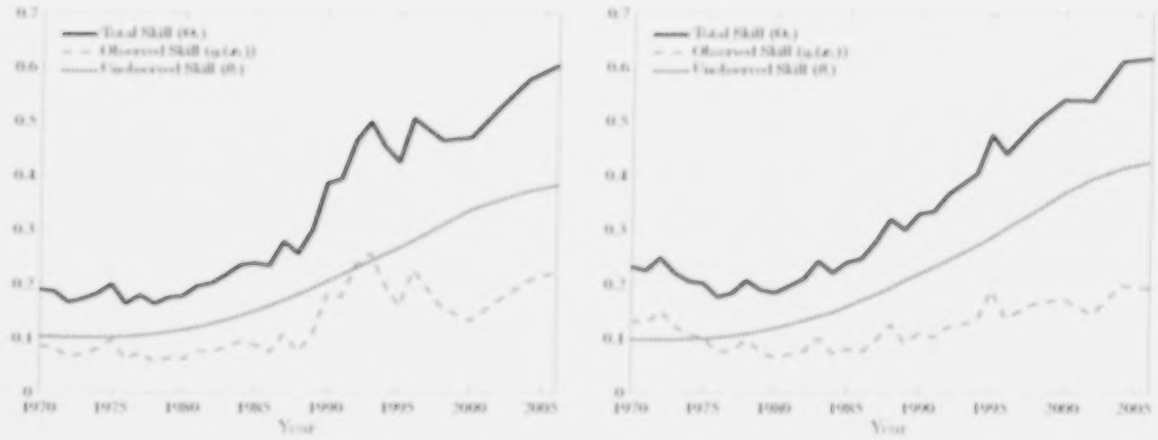
which is equivalent to our empirical log wage equation (1). The variance of skills consists of variation in both the observable and unobservable components. It is easily identified as follows:

$$\sigma(\Theta_t)^2 = \text{Var}(\Theta_t) = \frac{\text{Var}(\ln W_t) - \text{Var}(\varepsilon_t)}{\mu_t^2}.$$

Therefore, our estimates of μ_t and $\text{Var}(\varepsilon_t)$ can be used to compute the variance of skills in each period. Figure 20 shows the time trends in skill variances for college- and non-college-educated workers. These skill variances increased steeply from the early 1980s to the mid-2000s for both college- and non-college-educated workers, with most of the increase driven by unobserved skills.

Demand Factors Recovering output elasticities from factor income shares is not uncommon. However, implementing this strategy in our case is complicated by the need to obtain education sector-specific labor shares. The difficulty arises from the fact that data on value added by workers with different levels of education are not generally available. To overcome this difficulty, we assume that the labor shares in our model can be approximated by average industry-level labor share for workers in each sector. Suppose that there are $j = 1, \ldots, J$ industries with value added $V_{j,t}$ and labor compensation $L_{j,t}$ in year t. Let $N_{j,s,t}$ be

[44] It is worth noting, however, that the wage returns to skill, $d\ln W_t(\Theta_t)/d\Theta_t$, are distinct from and need not equal the wage returns to jobs or tasks, $d\ln W_t(\hat{\Theta}_t(Z_t))/dZ_t = \gamma_t + \lambda_t\sigma(\Theta_t)/\sigma(Z_t)$. Thus, it is possible for the returns to skill to decline when the returns to job productivity increase.



(a) Non-College (b) College

Figure 20: Skill Variances

the number of workers in industry j, sector s, and year t, and let $N_{s,t} = \sum_{j=1}^{J} N_{j,s,t}$ be the total number of workers in sector s and year t in all industries. Then, the average labor share in sector s and year t is

$$\sum_{j=1}^{J} \frac{N_{j,s,t}}{N_{s,t}} \frac{I_{\cdot j,t}}{V_{j,t}}.$$

We use data on labor shares and the number of college- and non-college-educated workers in the U.S. by 31 International Standard Industrial Classification of All Economic Activities (ISIC) [45] industries from the World KLEMS data set.[46] The number of workers is computed by counting male workers aged 25–64. Figure 21 shows that the implied labor shares exhibit weak long-run time trends and strongly co-move over time. The labor share is slightly lower for college workers until the late 1990s, but the gap closes thereafter.

Figure 22 shows the demand factors calculated from the labor shares and returns to skills. The time patterns for the partial elasticity of output with respect to skill, λ_t, are similar to patterns for the returns to skills, μ_t, due to the relatively modest time variation in labor shares. For both college- and non-college-educated workers, λ_t increases between the mid-1970s and the mid-1980s, then decreases until the early 2000s. The rise and (especially) fall of λ_t is much more pronounced for non-college-educated workers. The time pattern for the other demand factor, $\gamma_t \sigma(Z_t)$, is quite different from that of λ_t: During the 1970s, the partial elasticity of output to (scaled) job productivity increases only for non-college-educated

[45] ISIC is a United Nations industry classification system.

[46] See Jorgenson, Ho, and Samuels (2012) for description.

Figure 21: Average Labor Share

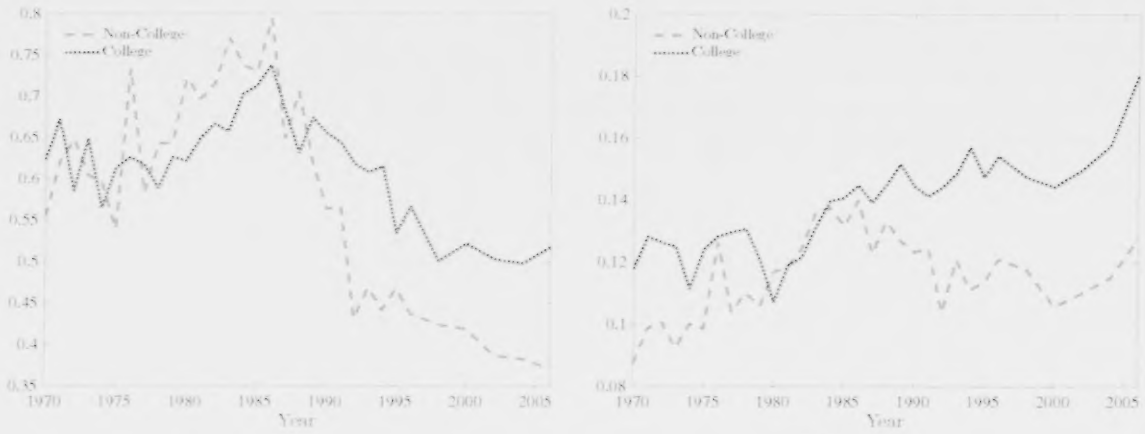
workers. Beginning in the early 1980s, this elasticity begins to increase for college-educated workers, while it begins to fall for non-college-educated workers.[47]

Effects of Demand and Supply Factors on Returns to Skill Finally, we assess the contribution of demand and supply factors to the evolution of returns to skill. Figure 23 shows the actual estimated returns, as well as counterfactual returns when the demand factors (λ_t and $\gamma_t \sigma(Z_t)$) or the supply factor ($\sigma(\Theta_t)$) are held constant at their 1985 values. While the patterns are qualitatively similar across the non-college and college sectors, the relative importance of supply and demand factors is not. Among non-college-educated men, the decline in returns to skill is driven almost exclusively by a reduction in demand for skill. While the increase in the variance of skills did contribute to the declining returns, its contribution is quite modest. Among college-educated men, both supply and demand factors contribute roughly equally to the decline in returns to skill between the mid-1980s and mid-2000s. Interestingly, for both college- and non-college-educated workers, the modest rise in skill returns over the 1970s and early 1980s is due entirely to increasing demand for skill.

7 Conclusion

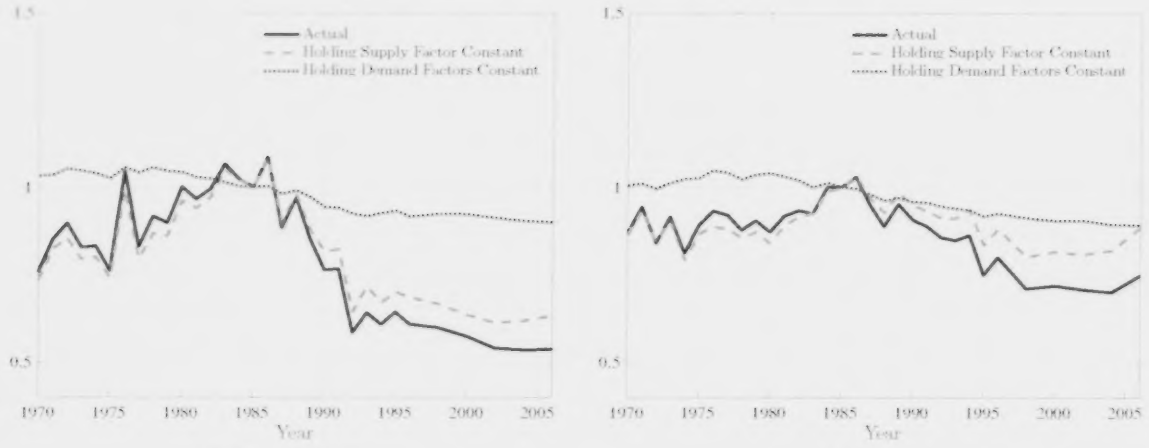
Economists have struggled to determine the underlying causes of rising wage inequality over the past few decades. Most efforts have relied on large repeated samples of cross-sectional data, attributing the

[47]The patterns for $\gamma_t \sigma(Z_t)$ are broadly consistent with an increase in wage returns to more productive jobs among college-educated workers and declines in wage returns to better jobs among non-college-educated workers as highlighted by the "polarization" literature.



(a) λ_t (b) $\gamma_t \sigma(Z_t)$

Figure 22: Demand Factors



(a) Non-College (b) College

Figure 23: Actual and Counterfactual Returns (μ_t)

growth in residual inequality to rising returns to unobserved skill while assuming that the distribution of unobserved skills has remained constant over time. More recent studies have often attempted to incorporate additional measures of worker skills or job tasks, continuing to rely on data drawn from different samples of workers over time. While these efforts have yielded important insights, they are not without limitations.

This paper takes a very different approach, demonstrating that traditional panel data sets can be used

to separately identify changes in the returns to unobserved skill from changes in the distributions of unobserved skill and in the distribution of transitory non-skill shocks. Based on transparent identifying assumptions, we show that a simple 2SLS strategy can be used to estimate the returns to unobserved skill over time, even when life cycle skill growth varies across individuals due to systematic unobserved heterogeneity and idiosyncratic shocks. Once skill returns have been identified, it is straightforward to identify and estimate the evolution of skill (and skill growth) distributions as well as distributions of transitory non-skill shocks. None of this requires measuring the tasks workers perform or efforts to directly measure worker skill levels.

Using panel data on the wages of American men from the PSID, we show that accounting for changes in the distributions of skills and the volatility of wages is critical in estimating the evolution of returns to unobserved skills. Our estimates reveal that these returns were fairly stable or increasing in the 1970s and early 1980s, but then fell sharply after 1985, especially among non-college-educated workers. The decline in returns was offset by a strong increase in the variance of unobserved skill beginning in the early 1980s, driven by increasing variation in life cycle skill growth. These conclusions stand in stark contrast to the prevailing view, which attributes rising residual inequality to rising returns.

To understand why the returns to skill have fallen since the mid-1980s, we develop an assignment model of the labor market in which workers of heterogeneous skill levels are matched to different jobs. We further show conditions under which this framework produces equilibrium wage functions like those commonly assumed in the empirical literature on wage inequality. Combining labor shares with our estimated skill returns and skill distributions, we identify changes in production technology and decompose changes in skill returns into supply vs. demand effects. Our estimates suggest that the fall in demand for skill explains most of the decline in returns for non-college-educated men, while both supply and demand shifts are similarly important for college-educated men.

Several of our findings are broadly consistent with the growing polarization literature, while highlighting the challenges faced by the simplest SBTC hypothesis (e.g., Card and DiNardo, 2002), which argues that a steady increase in demand for skills has driven the increase in inequality. Further work is required to understand why skill demand declined during the 1990s, precisely when technological progress appears to have accelerated (Cummins and Violante, 2002).

Equally important, our results suggest that more attention should be devoted to understanding the dramatic increase in unobserved skill inequality, stemming largely from growing differences in life cycle (post-school) skill growth across workers with similar experience and education levels. This may simply reflect a different type of technological change – one characterized by the frequent introduction of new tasks that makes others obsolete (Andolfatto and Smith, 2001; Acemoglu and Restrepo, 2017).[48] Defining

[48] In an economy with imperfect information about worker skills, our estimated "skill distributions" would instead reflect the distributions of beliefs about worker skills. What we refer to as "skill growth" would instead reflect updates in beliefs.

workers' skill levels by the most productive task(s) they can perform. this type of technological change would generate growing volatility in skills over the life cycle. Alternatively, more able workers may simply be more capable of learning and adapting to new tasks (Nelson and Phelps, 1966), which would lead to greater variation in systematic life cycle wage growth.[49] We do not attempt to explain the underlying causes for the growing skill growth inequality, leaving this for future research.

With this interpretation, the underlying distribution of skills may not have changed over time, but the market may have become better at identifying (and rewarding) skill differences (Lemieux, MacLeod, and Parent, 2009; Jovanovic, 2014).

[49]See Section 3.2 of Hornstein, Krusell, and Violante (2005) for a survey of theory and evidence on this view of technological change and skills.

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Appendix

A Identification Results

A.1 Identification with $\varepsilon_{i,t} \sim$ ARMA(1,q)

We demonstrate identification for the model in Section 2.2.1, generalized so that the transitory component $\varepsilon_{i,t}$ follows an ARMA(1,q) process. That is, $\varepsilon_{i,t} = \xi_{i,t}$ for $t = c_i + 1$ and, for $t > c_i + 1$,

$$\varepsilon_{i,t} = \rho\varepsilon_{i,t-1} + \sum_{j=0}^{\min\{q,t-c_i-1\}} \beta_j \xi_{i,t-j},$$

where $\beta_0 = 1$.

Identification of ρ Let $k = q + 1$. Then for (c, t, t'), such that $c < t \leq t' + k$,

$$\text{Cov}(\varepsilon_t, \varepsilon_{t'} - \rho\varepsilon_{t'-1}|c) = \text{Cov}\left(\varepsilon_t, \sum_{j=0}^{\min\{q,t-c-1\}} \beta_j \xi_{t'-j} \middle| c\right) = 0, \quad \text{and}$$

$$\text{Cov}(w_t, w_{t'}|c) - \rho\text{Cov}(w_t, w_{t'-1}|c) = \text{Cov}(w_t, w_{t'} - \rho w_{t'-1}|c) = \mu_t(\mu_{t'} - \rho\mu_{t'-1})\,\text{Var}(\theta_t|c).$$

Taking the ratio of this expression for cohort c relative to $\tilde{c} < t$ yields

$$\frac{\text{Cov}(w_t, w_{t'}|c) - \rho\text{Cov}(w_t, w_{t'-1}|c)}{\text{Cov}(w_t, w_{t'}|\tilde{c}) - \rho\text{Cov}(w_t, w_{t'-1}|\tilde{c})} = \frac{\text{Var}(\theta_t|c)}{\text{Var}(\theta_t|\tilde{c})}.$$

Similarly, for $t'' - t \geq k$,

$$\frac{\text{Cov}(w_t, w_{t''}|c) - \rho\text{Cov}(w_t, w_{t''-1}|c)}{\text{Cov}(w_t, w_{t''}|\tilde{c}) - \rho\text{Cov}(w_t, w_{t''-1}|\tilde{c})} = \frac{\text{Var}(\theta_t|c)}{\text{Var}(\theta_t|\tilde{c})}.$$

Combining these two equations yields

$$\frac{\text{Cov}(w_t, w_{t'}|c) - \rho\text{Cov}(w_t, w_{t'-1}|c)}{\text{Cov}(w_t, w_{t'}|\tilde{c}) - \rho\text{Cov}(w_t, w_{t'-1}|\tilde{c})} = \frac{\text{Cov}(w_t, w_{t''}|c) - \rho\text{Cov}(w_t, w_{t''-1}|c)}{\text{Cov}(w_t, w_{t''}|\tilde{c}) - \rho\text{Cov}(w_t, w_{t''-1}|\tilde{c})}. \tag{23}$$

Equation (23) can be written as

$$A\rho^2 + B\rho + C = 0, \tag{24}$$

where

$$\begin{aligned}
A &= \mathrm{Cov}(w_t, w_{t'-1}|c)\,\mathrm{Cov}(w_t, w_{t''-1}|\tilde{c}) - \mathrm{Cov}(w_t, w_{t''-1}|c)\,\mathrm{Cov}(w_t, w_{t'-1}|\tilde{c}),\\
B &= \mathrm{Cov}(w_t, w_{t'-1}|\tilde{c})\,\mathrm{Cov}(w_t, w_{t''}|c) + \mathrm{Cov}(w_t, w_{t'}|\tilde{c})\,\mathrm{Cov}(w_t, w_{t''-1}|c)\\
&\quad - \mathrm{Cov}(w_t, w_{t'}|c)\,\mathrm{Cov}(w_t, w_{t''-1}|\tilde{c}) - \mathrm{Cov}(w_t, w_{t'-1}|c)\,\mathrm{Cov}(w_t, w_{t''}|\tilde{c}),\\
C &= \mathrm{Cov}(w_t, w_{t'}|c)\,\mathrm{Cov}(w_t, w_{t''}|\tilde{c}) - \mathrm{Cov}(w_t, w_{t'}|\tilde{c})\,\mathrm{Cov}(w_t, w_{t''}|c).
\end{aligned}$$

If $A = 0$ or $B^2 - 4AC = 0$ holds for some $(c, \tilde{c}, t, t', t'')$ such that $c < t$, $\tilde{c} < t$, $t' - t \geq k$, and $t'' - t \geq k$, then ρ is identified from the unique solution to (24). Otherwise, there may be two real solutions (if $B^2 - 4AC > 0$).

When $A \neq 0$ and $B^2 - 4AC > 0$ holds for other available $(c, \tilde{c}, t, t', t'')$ such that $c < t$, $\tilde{c} < t$, $t' - t \geq k$, and $t'' - t \geq k$, the autocorrelation parameter ρ can be uniquely identified by combining at least two distinct quadratic equations (24). To see this, rewrite (24) for two different sets of $(c, \tilde{c}, t, t', t'')$:

$$\rho^2 + \frac{B_1}{A_1}\rho + \frac{C_1}{A_1} = 0, \tag{25}$$

$$\rho^2 + \frac{B_2}{A_2}\rho + \frac{C_2}{A_2} = 0. \tag{26}$$

If $A_1 \neq 0$, $B_1^2 - 4A_1C_1 > 0$, $A_2 \neq 0$, and $B_2^2 - 4A_2C_2 > 0$, then ρ is identified from the common solution to (25) and (26):

$$\rho = \left(\frac{B_1}{A_1} - \frac{B_2}{A_2}\right)^{-1}\left(\frac{C_2}{A_2} - \frac{C_1}{A_1}\right) = \frac{A_1C_2 - A_2C_1}{A_2B_1 - A_1B_2}$$

as long as $B_1/A_1 \neq B_2/A_2$.

Identification of μ_t For $t' - t \geq k$, suppose that there exists $(c, \tilde{c})$ such that $\text{Var}(\theta_{t-1}|c) \neq \text{Var}(\theta_{t-1}|\tilde{c})$ and $\text{Var}(\nu_t|c) = \text{Var}(\nu_t|\tilde{c})$. Then,

$$\begin{aligned}
&\frac{|\text{Cov}(w_t, w_{t'}|c) - \rho\text{Cov}(w_t, w_{t'-1}|c)| - |\text{Cov}(w_t, w_{t'}|\tilde{c}) - \rho\text{Cov}(w_t, w_{t'-1}|\tilde{c})|}{|\text{Cov}(w_{t-1}, w_{t'}|c) - \rho\text{Cov}(w_{t-1}, w_{t'-1}|c)| - |\text{Cov}(w_{t-1}, w_{t'}|\tilde{c}) - \rho\text{Cov}(w_{t-1}, w_{t'-1}|\tilde{c})|} \\
=&\frac{\mu_t(\mu_{t'} - \rho\mu_{t'-1})|\text{Var}(\theta_t|c) - \text{Var}(\theta_t|\tilde{c})|}{\mu_{t-1}(\mu_{t'} - \rho\mu_{t'-1})|\text{Var}(\theta_{t-1}|c) - \text{Var}(\theta_{t-1}|\tilde{c})|} \\
=&\frac{\mu_t}{\mu_{t-1}}
\end{aligned}$$

In this way, we can identify μ_t for $t \leq \bar{t} - k$.

For $t > \bar{t} - k$, consider $t' < t - k$. Then,

$$\frac{\text{Cov}(w_{t'}, w_t|c) - \rho\text{Cov}(w_{t'}, w_{t-1}|c)}{\text{Cov}(w_{t'}, w_{t-1}|c) - \rho\text{Cov}(w_{t'}, w_{t-2}|c)} = \frac{\mu_{t'}(\mu_t - \rho\mu_{t-1})\,\text{Var}(\theta_{t'}|c)}{\mu_{t'}(\mu_{t-1} - \rho\mu_{t-2})\,\text{Var}(\theta_{t'}|c)} = \frac{\mu_t - \rho\mu_{t-1}}{\mu_{t-1} - \rho\mu_{t-2}} \tag{27}$$

Since ρ and μ_t for $t \leq \bar{t} - k$ are identified, we can sequentially identify μ_t for $t > \bar{t} - k$ using the above equation.

Identification of $\text{Var}(\theta_t|c)$ For $t' - t \geq k$,

$$\text{Var}(\theta_t|c) = \frac{\text{Cov}(w_t, w_{t'}|c) - \rho\text{Cov}(w_t, w_{t'-1}|c)}{\mu_t(\mu_{t'} - \rho\mu_{t'-1})}.$$

Therefore, $\text{Var}(\theta_t|c)$ is identified for all $t \leq \bar{t} - k$.

Identification of β_j First, note that the ARMA$(1,q)$ process can be written as an MA$(t - c + 1)$ process:

$$\varepsilon_{it} = \sum_{j=0}^{t-c_i-1} \tilde{\beta}_j \xi_{it-j},$$

where $\tilde{\beta}_j = 1$ for $j = 0$, $\tilde{\beta}_j = \rho\tilde{\beta}_{j-1} + \beta_j$ for $1 \leq j \leq q$, and $\tilde{\beta}_j = \rho\tilde{\beta}_{j-1}$ for $j > q$.

Then, for $t = c + 1$ and $j \geq 0$,

$$\frac{\text{Cov}(\varepsilon_t, \varepsilon_{t+j}|c)}{\text{Var}(\varepsilon_t|c)} = \tilde{\beta}_j \frac{\text{Var}(\xi_t|c)}{\text{Var}(\xi_t|c)} = \tilde{\beta}_j.$$

Therefore, $\tilde{\beta}_j$'s are identified from cohort-specific autocovariances of ε_t, which can be obtained from

$$\text{Cov}(\varepsilon_t, \varepsilon_{t+j}|c) = \text{Cov}(w_t, w_{t+j}|c) - \mu_t\mu_{t+j}\,\text{Var}(\theta_t|c).$$

Given ρ and $\tilde{\beta}_j$, β_j for $1 \leq j \leq q$ is identified from $\beta_j = \tilde{\beta}_j - \rho\tilde{\beta}_{j-1}$.

Identification of $\text{Var}(\xi_t|c)$ For the initial period $t = c + 1$ for cohort c, $\text{Var}(\xi_t|c) = \text{Var}(\varepsilon_t|c)$. For $c + 1 < t \leq \bar{t} - k$,

$$\text{Var}(\xi_t|c) = \text{Var}(\varepsilon_t|c) - \sum_{j=1}^{\min\{q, t-c-1\}} \tilde{\beta}_j^2 \, \text{Var}(\xi_{t-j}|c).$$

A.2 Identification with Heterogeneous Skill Growth Rates

We demonstrate identification for the model in Section 2.2.2 with systematic heterogeneity in life cycle skill growth:

$$\theta_{i,t} = \theta_{i,t-1} + \tau_t(c_i)\delta_i + \tilde{v}_{i,t},$$

where $\tau_t(c) = 0$ for $e = t - c \geq \overline{e}$.

Identification of μ_t Note that μ_t/μ_{t-1} for $t > \underline{t} + k$ is identified if there exists some cohort c such that (i) the cohort has experience $e = t' - c \geq \overline{e}$ in some year $t' < t - k$ and (ii) the cohort is observed in years t', $t - 1$, and t. These require that $\overline{e} + k < 40$, which holds under our assumption $\overline{e} = 30$ and $k = 6$.

Moreover, μ_t/μ_{t-1} for $t \leq \underline{t} + k$ is identified if there exist two cohorts c and $\tilde{c}$ such that (i) both cohorts have experience of at least $\overline{e}$ in year $t - 1$, (ii) both cohorts are observed in years $t - 1$, t, and some year $t' \geq t + k$, and (iii) $\text{Var}(\theta_{t-1}|c) \neq \text{Var}(\theta_{t-1}|\tilde{c})$ and $\text{Var}(v_t|c) = \text{Var}(v_t|\tilde{c})$. For the first two conditions to be satisfied, we need $\overline{e} + k < 39$.

Identification of $\tau_t(c)$ By dividing the residual by μ_t, we get

$$\frac{w_{i,t}}{\mu_t} = \theta_{i,t} + \frac{\varepsilon_{i,t}}{\mu_t}.$$

If we take a first difference,

$$\Delta\left(\frac{w_{i,t}}{\mu_t}\right) = \Delta\theta_{i,t} + \Delta\left(\frac{\varepsilon_{i,t}}{\mu_t}\right) = \tau_t(c_i)\delta_i + \tilde{v}_{i,t} + \Delta\left(\frac{\varepsilon_{i,t}}{\mu_t}\right).$$

For (c, t, t') such that $\text{Cov}(\Delta\varepsilon_t, \Delta\varepsilon_{t'}|c) = \text{Cov}(\Delta\varepsilon_{t-1}, \Delta\varepsilon_{t'}|c) = 0$, we have

$$\text{Cov}\left(\Delta\left(\frac{w_{t-1}}{\mu_{t-1}}\right), \Delta\left(\frac{w_{t'}}{\mu_{t'}}\right)\middle|c\right) = \text{Cov}(\Delta\theta_{t-1}, \Delta\theta_{t'}|c) = \tau_{t-1}(c)\tau_{t'}(c)\,\text{Var}(\delta|c)$$

$$\text{Cov}\left(\Delta\left(\frac{w_t}{\mu_t}\right), \Delta\left(\frac{w_{t'}}{\mu_{t'}}\right)\middle|c\right) = \text{Cov}(\Delta\theta_t, \Delta\theta_{t'}|c) = \tau_t(c)\tau_{t'}(c)\,\text{Var}(\delta|c).$$

By combining these two, we can identify changes in $\tau_t(c)$:

$$\frac{\tau_t(c)}{\tau_{t-1}(c)} = \frac{\text{Cov}\left(\Delta\left(\frac{w_t}{\mu_t}\right), \Delta\left(\frac{w_{t'}}{\mu_{t'}}\right)|c\right)}{\text{Cov}\left(\Delta\left(\frac{w_{t-1}}{\mu_{t-1}}\right), \Delta\left(\frac{w_{t'}}{\mu_{t'}}\right)|c\right)}.$$

Identification of $\text{Var}(\delta|c)$ Once $\tau_t(c)$'s have been identified (up to a normalization $\tau_{t^*(c)}(c) = 1$ for some $t^*(c)$), $\text{Var}(\delta|c)$ is also identified from

$$\text{Var}(\delta|c) = \frac{\text{Cov}\left(\Delta\left(\frac{w_t}{\mu_t}\right), \Delta\left(\frac{w_{t'}}{\mu_{t'}}\right)|c\right)}{\tau_t(c)\tau_{t'}(c)}.$$

Identification of $\text{Cov}(\psi, \delta|c)$ For (c, t, t') such that $t' - t \geq k + 1$, $\text{Cov}(\varepsilon_t, \Delta\varepsilon_{t'}|c) = 0$ and we get

$$\text{Cov}\left(\frac{w_t}{\mu_t}, \Delta\left(\frac{w_{t'}}{\mu_{t'}}\right)\Big|c\right) = \text{Cov}(\theta_t, \Delta\theta_{t'}|c) = \tau_{t'}(c)\,\text{Cov}(\theta_t, \delta|c),$$

where

$$\text{Cov}(\theta_t, \delta|c) = \text{Cov}(\psi, \delta|c) + \text{Var}(\delta|c) \sum_{j=0}^{t-c-1} \tau_{t-j}(c). \tag{28}$$

Therefore,

$$\text{Cov}(\psi, \delta|c) = \frac{\text{Cov}\left(\frac{w_t}{\mu_t}, \Delta\left(\frac{w_{t'}}{\mu_{t'}}\right)|c\right)}{\tau_{t'}(c)} - \text{Var}(\delta|c) \sum_{j=0}^{t-c-1} \tau_{t-j}(c).$$

Identification of $\text{Var}(\theta_t|c)$ For (c, t, t') such that $t' - t \geq k$, write

$$\theta_{i,t'} = \theta_{i,t} + \sum_{j=0}^{t'-t-1} \left[\tau_{t'-j}(c_i)\delta_i + \tilde{v}_{i,t'-j}\right].$$

Then,

$$\text{Cov}\left(\frac{w_t}{\mu_t}, \frac{w_{t'}}{\mu_{t'}}\Big|c\right) = \text{Cov}(\theta_t, \theta_{t'}|c) = \text{Var}(\theta_t|c) + \text{Cov}(\theta_t, \delta|c) \sum_{j=0}^{t'-t-1} \tau_{t'-j}(c).$$

Therefore,

$$\text{Var}(\theta_t|c) = \text{Cov}\left(\frac{w_t}{\mu_t}, \frac{w_{t'}}{\mu_{t'}}\Big|c\right) - \text{Cov}(\theta_t, \delta|c) \sum_{j=0}^{t'-t-1} \tau_{t'-j}(c).$$

Identification of $\text{Var}(\tilde{\nu}_t|c)$ Note that

$$\text{Var}(\theta_{t+1}|c) = \text{Var}(\theta_t|c) + \text{Var}(\delta|c)\tau_{t+1}(c)^2 + 2\,\text{Cov}(\theta_t, \delta|c)\tau_{t+1}(c) + \text{Var}(\tilde{\nu}_{t+1}|c).$$

Therefore,

$$\text{Var}(\tilde{\nu}_{t+1}|c) = \text{Var}(\theta_{t+1}|c) - \text{Var}(\theta_t|c) - \text{Var}(\delta|c)\tau_{t+1}(c)^2 - 2\,\text{Cov}(\theta_t, \delta|c)\tau_{t+1}(c).$$

B GMM Estimates of Skill Returns, Overidentification Tests, and Variance of Skill Growth

In this appendix, we report GMM estimates for the returns to skill using the same model and moments (i.e., lagged residuals serve as instruments) as with our 2SLS approach in Section 4 along with J-statistics to test for overidentification. We also report analogous GMM estimates that use both past and future wage residuals as instruments, reporting J-statistics to test the validity of the latter. Finally, we combine estimates using past vs. future residuals as instruments to estimate the variance of skill growth relative to lagged skill levels.

To begin, rewrite the two-period wage growth equation (12) as follows:

$$w_{i,t} - w_{i,t-2} = \left(\frac{\mu_t - \mu_{t-2}}{\mu_{t-2}}\right) w_{i,t-2} + u_{i,t}, \tag{29}$$

where $u_{i,t} = \varepsilon_{i,t} - \frac{\mu_t}{\mu_{t-2}}\varepsilon_{i,t-2} + \mu_t(\nu_{i,t-1} + \nu_{i,t})$.

Our model with skill serially uncorrelated skill shocks (Section 2.2.1) implies the following moment condition:

$$\text{E}[w_{t'} u_t] = 0, \quad \text{for } t' \leq t - 2 - k. \tag{30}$$

Under the stronger assumption that $\text{Var}(\nu_t) = 0$ for all t, the following additional moment condition holds:

$$\text{E}[w_{t''} u_t] = 0, \quad \text{for } t'' \geq t + k. \tag{31}$$

Equation (31) will not hold when $\text{Var}(\nu_{t-1}) + \text{Var}(\nu_t) > 0$, and the IV estimate using future residuals as

instruments is asymptotically biased with probability limit

$$\frac{\text{Cov}(w_t - w_{t-2}, w_{t'})}{\text{Cov}(w_{t-2}, w_{t'})} = \left(\frac{\mu_t - \mu_{t-2}}{\mu_{t-2}}\right) + \frac{\mu_t}{\mu_{t-2}}\left(\frac{\text{Var}(v_{t-1}) + \text{Var}(v_t)}{\text{Var}(\theta_{t-2})}\right) > \frac{\mu_t - \mu_{t-2}}{\mu_{t-2}}, \qquad \text{for } t' \geq t + k.$$

The difference between estimates using future and past residuals as instruments identifies the magnitude of the skill shock variance relative to the skill variance: for $t' \leq t - 2 - k$ and $t'' \geq t + k$,

$$\frac{\text{Var}(v_{t-1}) + \text{Var}(v_t)}{\text{Var}(\theta_{t-2})} = \left(\frac{\text{Cov}(w_t - w_{t-2}, w_{t''})}{\text{Cov}(w_{t-2}, w_{t''})} - \frac{\text{Cov}(w_t - w_{t-2}, w_{t'})}{\text{Cov}(w_{t-2}, w_{t'})}\right)\left(1 + \frac{\text{Cov}(w_t - w_{t-2}, w_{t'})}{\text{Cov}(w_{t-2}, w_{t'})}\right)^{-1}. \tag{32}$$

B.1 Overidentification Tests

We begin by testing the moments in equation (31) using Hansen's J-test, assuming $k = 6$ and using the two nearest valid instruments. This amounts to using $w_{i,t-8}$ and $w_{i,t-9}$ (or $w_{i,t-10}$) for equation (30) and the first two available out of $w_{i,t+6}$, $w_{i,t+7}$, $w_{i,t+8}$, $w_{i,t+9}$ for (31).

Table B-1 reports the two-step optimal GMM estimates (allowing for heteroskedasticity and serial correlation within individual) for the coefficient on $w_{i,t-2}$ along with Hansen's J-statistics when estimating the wage growth equation (29). Panel A reports estimates when moments from both equations (30) and (31) are used (i.e., lags and leads), while Panel B reports estimates when only the moment condition from equation (30) is used (i.e., lags only). The sample is restricted to be the same in both panels.[50]

Comparing the J-statistics in Panels A and B in Table B-1, we can test the validity of using leads as instruments (i.e., moments in equation (31)). Since the differences are greater than 5.991 (critical value for χ^2_2 at significance level 0.05) except for 1979–1980 and 2002–2004, we reject the "leads" moments in equation (31) at 5% significance level for 1981-2000. Moreover, all J-statistics in Panel B are smaller than 3.841 (critical value for χ^2_1 at significance level 0.05), implying that we cannot reject the lags as instruments (i.e., moments in equation (30)) at the 5% level. Altogether, these results suggest that the lagged residuals are valid instruments, while the leads are not (in most years).

Also note that the estimates using both leads and lags as instruments are always greater than their counterparts using only the lags, consistent with the positive bias induced from using leads when there are skill growth shocks.

[50]Because use of both leads and lags requires observations that are as many as 19 years apart, this restriction reduces the sample size substantially relative to that used in our baseline 2SLS analysis (see Tables 1 and 2). Panel A of Table B-2 below reports GMM estimates when this sample selection is not imposed. Those results are directly comparable and quite similar to those in Tables 1 and 2.

Table B-1: GMM Estimates of Skill Return Growth Using Leads and Lags as Instruments (Balanced Samples)

	1979–80	1981–83	1984–86	1987–89	1990–92	1993–95	1996–2000	2002–04
A. 2 Nearest Valid Lags and 2 Nearest (Potentially Valid) Leads as Instruments								
Coeff. on $w_{i,t-2}$	-0.019	0.088*	0.053	0.007	-0.030	0.026	0.008	0.022
	(0.053)	(0.044)	(0.046)	(0.034)	(0.038)	(0.035)	(0.0235)	(0.035)
Observations	818	1,251	1,325	1,356	1,313	1,311	1,375	777
J-Statistic	4.400	10.392	11.743	9.579	9.461	6.991	8.922	1.646
B. 2 Nearest Valid Lags as Instruments								
Coeff. on $w_{i,t-2}$	-0.070	-0.010	-0.065	-0.057	-0.103*	-0.025	-0.041	-0.003
	(0.056)	(0.053)	(0.055)	(0.040)	(0.046)	(0.039)	(0.029)	(0.0389)
Observations	818	1,251	1,325	1,356	1,313	1,311	1,375	777
J-Statistic	0.009	0.187	0.632	0.869	0.064	0.238	0.107	0.016

Notes: GMM estimates for a regression of $(w_{i,t} - w_{i,t-2})$ on $w_{i,t-2}$. Panel A uses as instruments the 2 nearest available lags from $(w_{t-8}, w_{t-9}, w_{t-10})$ and 2 nearest available leads from $(w_{t+6}, ..., w_{t+9})$. Panel B uses only the 2 lags as instruments. * Denotes significance at 0.05 level.

B.2 Inferring Relative Magnitude of Skill Shocks

Table B-2 reports GMM estimates using only lags or leads as instruments where all available observations are used (i.e., samples are not restricted to be the same across specifications). Panel A reports estimates when only the moments in equation (30) are used (i.e., 2 nearest valid lags). These results are analogous to the 2SLS estimates in Tables 1 and 2, using the same samples. Comparing estimates across the tables, we see that they are quite similar. Panel B reports GMM estimates when only the moments in equation (31) are used (i.e., 2 nearest potentially valid leads), also based on all available observations. Finally, we compare the estimates in Panels A and B using equation (32) to estimate the relative importance of skill growth shocks. These estimates are reported in Panel C. The variance of (two-year) skill growth relative to the variance of prior skill levels ranges from 0.16 to 0.29 over our entire sample period.

Table B-2: GMM Estimates of Skill Return Growth Using Leads vs. Lags as Instruments and Relative Skill Shock Variance (Unbalanced Samples)

	1979–80	1981–83	1984–86	1987–89	1990–92	1993–95	1996–2000	2002–04
A. 2 Nearest Valid Lags as Instruments								
Coeff. on $w_{i,t-2}$	-0.033	-0.045	-0.044	-0.084*	-0.083*	-0.067	-0.076*	-0.090*
	(0.045)	(0.038)	(0.038)	(0.033)	(0.035)	(0.035)	(0.025)	(0.035)
Observations	1,349	2,077	2,188	2,245	2,189	2,095	2,122	1,377
B. 2 Nearest (Potentially Valid) Leads as Instruments								
Coeff. on $w_{i,t-2}$	0.165*	0.229*	0.193*	0.099*	0.067	0.087*	0.073*	0.115*
	(0.059)	(0.053)	(0.047)	(0.042)	(0.043)	(0.038)	(0.028)	(0.039)
Observations	1,500	2,229	2,159	2,100	2,042	1,994	2,178	1,249
C. Estimated Shock Variances Relative to Skill Variances								
$\frac{\mathrm{Var}(v_{t-1})+\mathrm{Var}(v_t)}{\mathrm{Var}(\theta_{t-2})}$	0.204	0.287	0.248	0.200	0.163	0.166	0.161	0.225

Notes: GMM estimates for a regression of $(w_{i,t} - w_{i,t-2})$ on $w_{i,t-2}$. Panel A uses 2 nearest available lags as instruments from $(w_{t-8}, w_{t-9}, w_{t-10})$. Panel B uses 2 nearest available leads as instruments from $(w_{t+6}, ..., w_{t+9})$. Panel C reports estimates of skill growth shock variance relative to skill variance based on equation (32).

* Denotes significance at 0.05 level.

C Proofs and Analytical Details for Assignment Model

C.1 Proof of Proposition 2

With the normality assumption, the market clearing condition (18) simplifies to

$$\hat{Z}_t(\Theta_t) = \mathrm{E}[Z_t] + \frac{\sigma(Z_t)}{\sigma(\Theta_t)}\left(\Theta_t - \mathrm{E}[\Theta_t]\right). \tag{33}$$

Then the first-order condition (17) becomes

$$\frac{dW_t(\Theta_t)}{d\Theta_t} = \lambda_t \exp\left(\lambda_t\Theta_t + \gamma_t\hat{Z}_t(\Theta)\right) = \lambda_t \exp\left(\gamma_t\left(\mathrm{E}[Z_t] - \frac{\sigma(Z_t)}{\sigma(\Theta_t)}\mathrm{E}[\Theta_t]\right) + \left(\lambda_t + \gamma_t\frac{\sigma(Z_t)}{\sigma(\Theta_t)}\right)\Theta_t\right).$$

By integrating the above equation, we get

$$W_t(\Theta_t) = \int_{-\infty}^{\Theta_t}\frac{dW_t(\Theta_t')}{d\Theta_t}d\Theta_t' = \left(\frac{\lambda_t}{\lambda_t + \gamma_t\frac{\sigma(Z_t)}{\sigma(\Theta_t)}}\right)\exp\left(\gamma_t\left(\mathrm{E}[Z_t] - \frac{\sigma(Z_t)}{\sigma(\Theta_t)}\mathrm{E}[\Theta_t]\right) + \left(\lambda_t + \gamma_t\frac{\sigma(Z_t)}{\sigma(\Theta_t)}\right)\Theta_t\right).$$

By taking logs and differentiating with respect to Θ_t, we get (20).

D Calculating Standard Errors

Let $m = 1, 2, \ldots, M$ be the index of moments. Let $d_{i,m}$ be the indicator of whether individual i contributes to the m^{th} moment $\text{Cov}(w_t, w_{t'}|s, E_j)$. That is, both $w_{i,t}$ and $w_{i,t'}$ are non-missing and $s_{i,t} = s_{i,t'} = s$ and $e_{i,t} \in E_j$. Also let $p_m(\mathbf{\Lambda}) = \text{Cov}(w_t, w_{t'}|s, E_j, \mathbf{\Lambda})$. Then we can write

$$h_m(z_i, \mathbf{\Lambda}) = d_{i,m}[w_{i,t}w_{i,t'} - p_m(\mathbf{\Lambda})],$$

where z_i includes $w_{i,t}$ $d_{i,m}$ for all t and m for individual i. Let $\boldsymbol{h}(z, \mathbf{\Lambda}) = [h_1(z, \mathbf{\Lambda})\, h_2(z, \mathbf{\Lambda})\, \ldots\, h_M(z, \mathbf{\Lambda})]^\top$. Then the following moment condition holds for the true parameter $\mathbf{\Lambda}_0$:

$$\text{E}[\boldsymbol{h}(z, \mathbf{\Lambda}_0)] = \mathbf{0}.$$

The minimum distance estimator $\hat{\mathbf{\Lambda}}$ is equivalent to the GMM estimator that solves

$$\min_{\mathbf{\Lambda}} \left[\frac{1}{N}\sum_{i=1}^{N} \boldsymbol{h}(z_i, \mathbf{\Lambda})\right]^\top \boldsymbol{W} \left[\frac{1}{N}\sum_{i=1}^{N} \boldsymbol{h}(z_i, \mathbf{\Lambda})\right],$$

where $\boldsymbol{W} = \text{diag}(\frac{N^2}{N_1^2}, \frac{N^2}{N_2^2}, \ldots, \frac{N^2}{N_M^2})$ and $N_m = \sum_{i=1}^{N} d_{i,m}$.

The GMM estimator $\hat{\mathbf{\Lambda}}$ is asymptotically normal with a variance matrix

$$\boldsymbol{V} = (\boldsymbol{H}^\top \boldsymbol{W} \boldsymbol{H})^{-1}(\boldsymbol{H}^\top \boldsymbol{W} \mathbf{\Omega} \boldsymbol{W} \boldsymbol{H})(\boldsymbol{H}^\top \boldsymbol{W} \boldsymbol{H})^{-1},$$

where $\boldsymbol{H}$ is the Jacobian of the vector of moments, $\text{E}[\partial \boldsymbol{h}(z, \mathbf{\Lambda}_0)/\partial \mathbf{\Lambda}^\top]$, and $\mathbf{\Omega} = \text{E}[\boldsymbol{h}(z, \mathbf{\Lambda}_0)\boldsymbol{h}(z, \mathbf{\Lambda}_0)^\top]$. Both expectations are replaced by sample averages and evaluated at the estimated parameter:

$$\hat{\boldsymbol{H}} = \frac{1}{N}\sum_{i=1}^{N} \frac{\partial \boldsymbol{h}(z_i, \hat{\mathbf{\Lambda}})}{\partial \mathbf{\Lambda}^\top} = \boldsymbol{W}^{-\frac{1}{2}} \frac{\partial \boldsymbol{p}(\hat{\mathbf{\Lambda}})}{\partial \mathbf{\Lambda}^\top},$$

$$\hat{\mathbf{\Omega}} = \frac{1}{N}\sum_{i=1}^{N} \boldsymbol{h}(z_i, \hat{\mathbf{\Lambda}})\boldsymbol{h}(z_i, \hat{\mathbf{\Lambda}})^\top,$$

where $\boldsymbol{W}^{-\frac{1}{2}} = \text{diag}(\frac{N_1}{N}, \frac{N_2}{N}, \ldots, \frac{N_M}{N})$.

We can test r linear parameter restrictions $H_0 : \boldsymbol{R}\mathbf{\Lambda} = 0$ using Wald test statistic:

$$N(\boldsymbol{R}\hat{\mathbf{\Lambda}})^\top (\boldsymbol{R}\hat{\boldsymbol{V}}\boldsymbol{R}^\top)^{-1}\boldsymbol{R}\hat{\mathbf{\Lambda}} \xrightarrow{d} \chi_r^2.$$

E Additional Empirical Results

To examine whether attrition affects the residual autocovariances reported in Figure 4, Figure E-1 shows the autocovariances, $\text{Cov}(w_b, w_t)$ for $6 \leq t - b \leq 16$, where the samples for each line (representing different base years, b) are restricted to those individuals observed in the base year as well as at least one of the last two years used for that line (i.e., $t - b = 15$ or 16 in early years or $t - b = 14$ or 16 in later years with biannual surveys). Comparing Figures 4 and E-1, the autocovariance patterns are quite similar, indicating little effect of sample attrition (due to non-response or retirement) on the key moments used in our analysis.

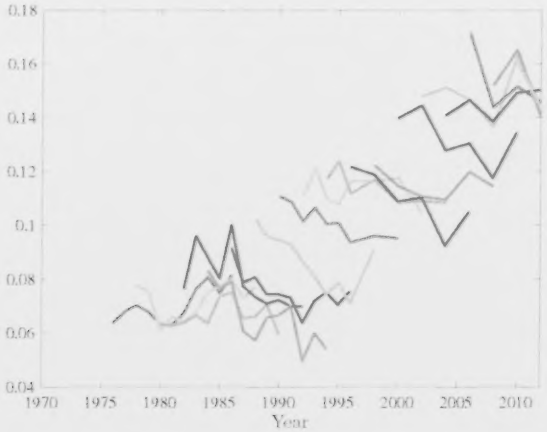


Figure E-1: Log Wage Residual Autocovariances ("Balanced" Sample)

Figure E-2 shows the residual autocovariances for individuals with 1–15 years of experience in the base years. Regardless of the base year, the autocovariances are typically declining from the late 1980s through the 1990s as in Figures 4 (full sample) and 5 (men with 16–30 years of experience) in the text. The lines also shift upwards over time, consistent with rising skill variances.

For the model estimated separately for college- and non-college-educated men in Section 5.2, Figures E-3 to E-6 report estimates (with 95% confidence intervals) for the variance of initial skills by cohort (Figure E-3), experience patterns for the variance of skill shocks (Figure E-4), and time trends (Figure E-5) and experience patterns (Figure E-6) for the variance of transitory non-skill shocks.

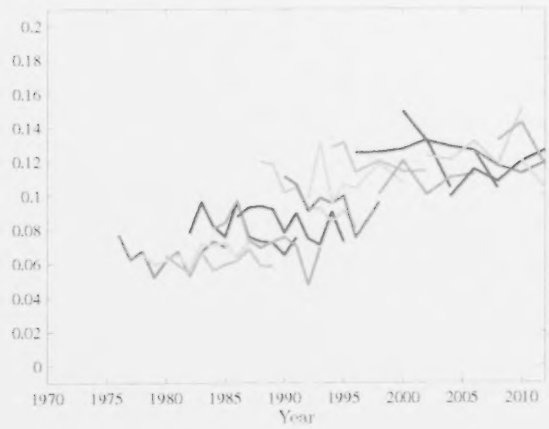
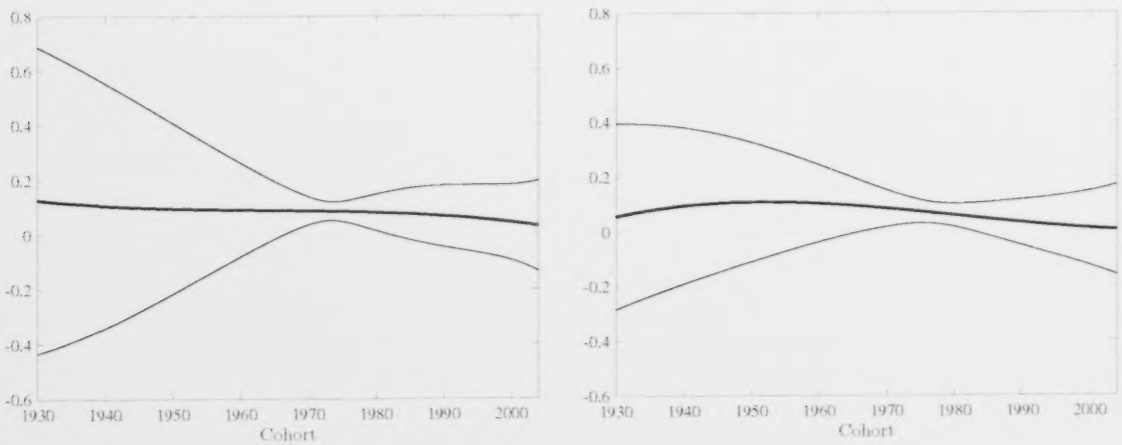


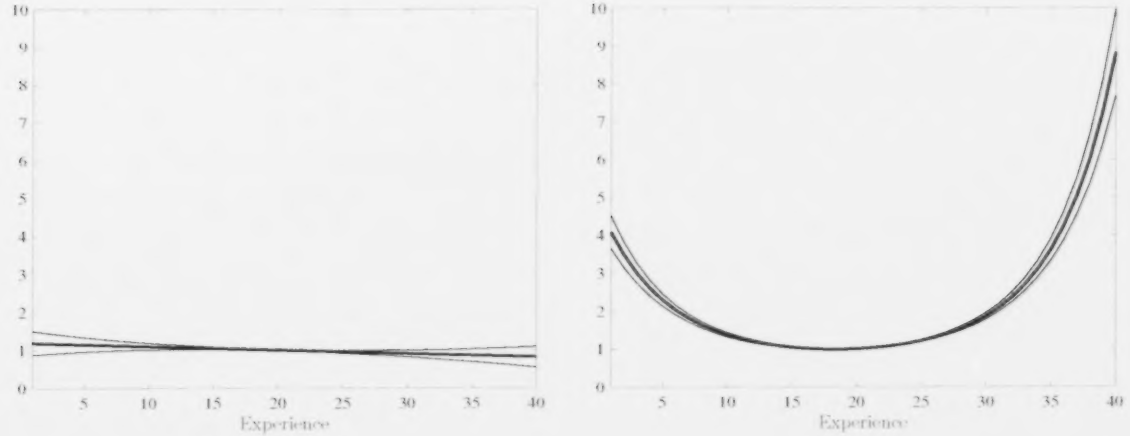
Figure E-2: Autocovariances for Log Wages (1–15 Years of Experience)



(a) Non-College

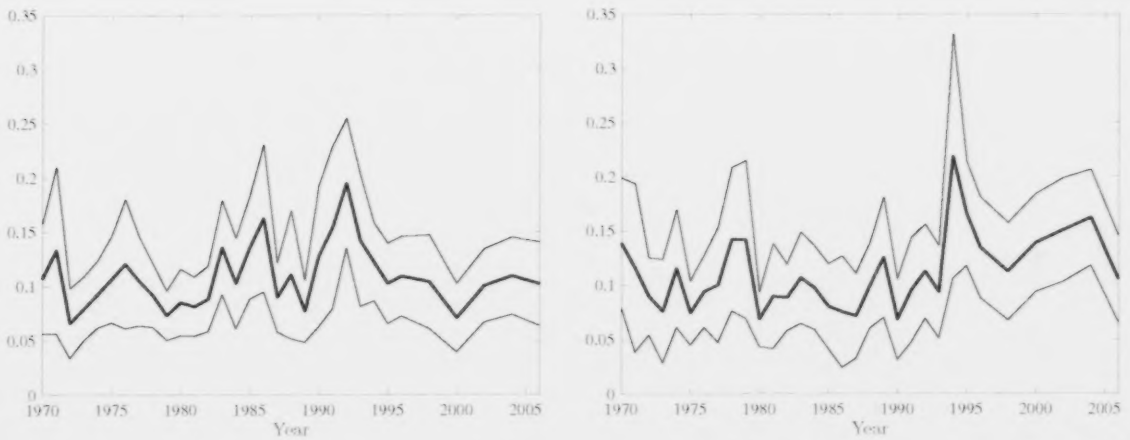
(b) College

Figure E-3: Variance of Initial Skill by Education



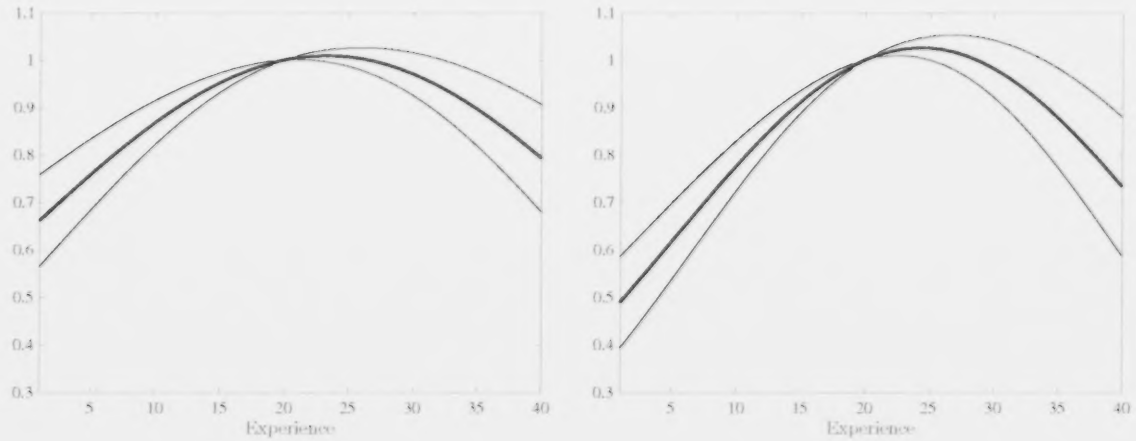
(a) Non-College (b) College

Figure E-4: Experience Patterns for the Variances of Skill Shocks, $\phi(e)$, by Education



(a) Non-College (b) College

Figure E-5: Time Trends in the Variances of Transitory Non-Skill Shocks, $\omega(t)$, by Education



(a) Non-College (b) College

Figure E-6: Experience Patterns for the Variances of Transitory Non-Skill Shocks, $\kappa(e)$, by Education